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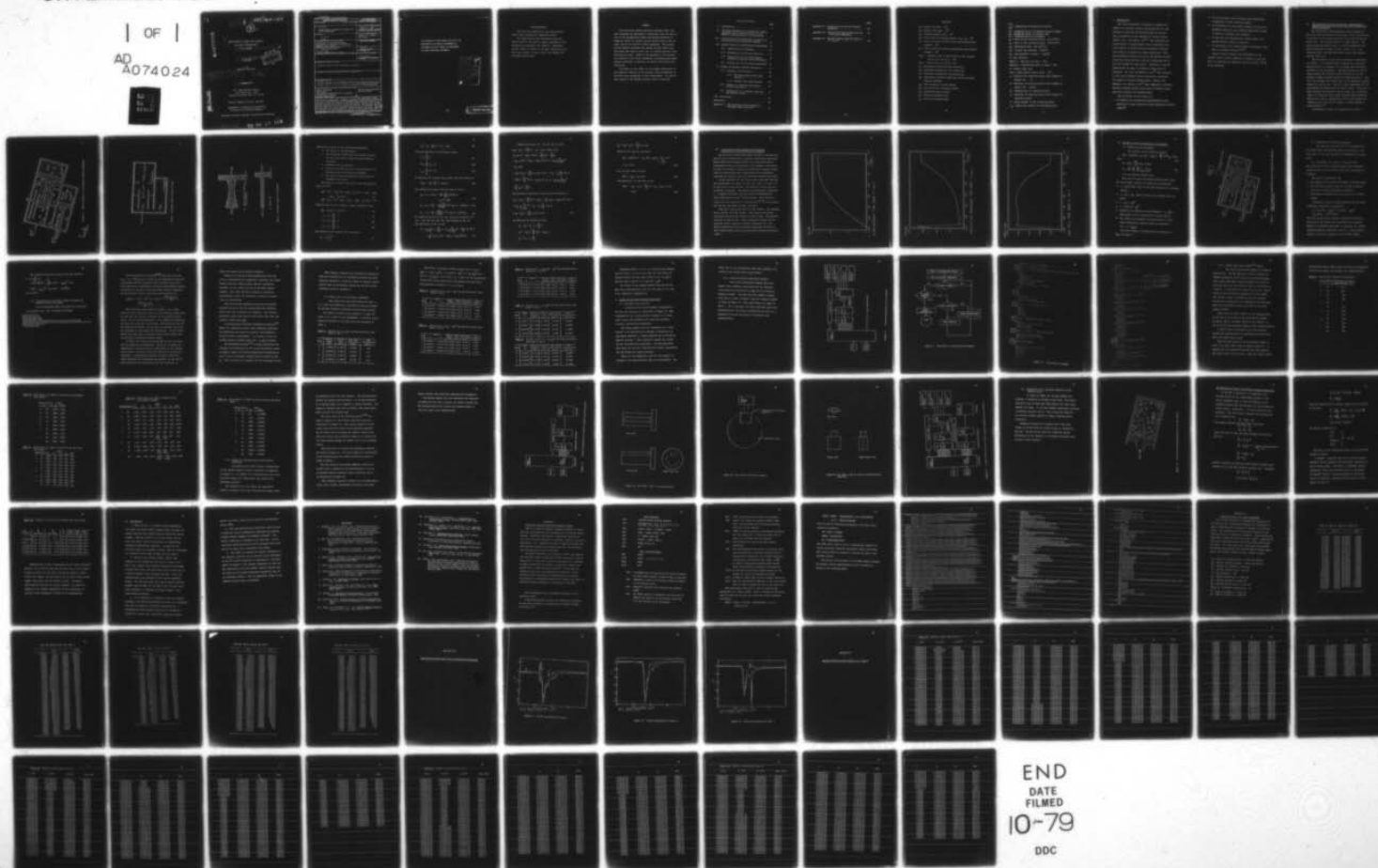
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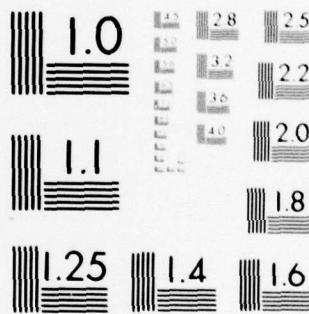
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SUMMARY

The Continuous System Simulation Language (CSSL) computer programs are developed in conjunction with the derivation of the mathematical models and optimum algorithm to study the effect of various orifice control laws on the optimal control for the M-37 recoil mechanism. The optimal control greatly decreases the average rod pull peak force and causes the force to level out to a nearly constant value following the peak. Based on the analysis of the acceleration signals of the recoil mechanism, a microprocessor-based digital controller is designed to control the orifice area controller.

The effect of the change in the thermal conductance of the composite material on the rod pull force performance of the M-37 recoil mechanism is also investigated. No significant change in the maximum rod pull force is observed.

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NOTATION

- A_R - Recoil rod area. in^2
- A_C - Control rod area. in^2
- A_D - Floating piston area. in^2
- A_N - Total area of floating piston = $A_C + A_D$. in^2
- A_1 - Area of orifice between recoil and recuperating chamber. in^2
- A_2 - Area of orifice between recuperating and control chamber. in^2
- $A_3(x)$ - Variable area of the groove in the floating piston at position x . in^2
- $B(t)$ - Breech force at time t . lbf
- C_1 - Discharge coefficient for orifice A_1 .
- C_2 - Discharge coefficient for orifice A_2 .
- C_3 - Discharge coefficient for orifice A_3 .
- C_q - Equivalent friction coefficient for frictional loss at orifices
- F_R - Dry friction at recoil piston.
- F_P - Dry friction at floating piston.
- F_q - Equivalent dry friction.
- g_1 - Position feedback gain.
- g_2 - Velocity feedback gain.

J_1 |
 J_2 | - Components of objective function

W_1 - weighting factor of maximum verticle length
 W_2 - weighting factor of orifice area
 W_3 - weighting factor of pressure
 m_p - Mass of the floating piston lbf sec²/in
 m_R - Mass of the recoiling parts lbf sec²/in
 m_q - Equivalent mass lbf sec²/in
 P_0 - Initial gas pressure lbf/in²
 P_G - Gas pressure at time t lbf/in²
 R - Gas constant
 $RDPL(t)$ - Rod pull at time t lbf
 $RDPLD(t)$ - Desired Rod pull at time t lbf
 T - Recoil time sec
 $U(t)$ - Open area of servo valve in²
 x - Position of recoiling parts with respect to
 recoil rod in
 \dot{x} - Velocity of recoiling parts with respect to
 recoil rod in/sec
 \ddot{x} - Acceleration of recoiling parts
 y - Position of floating piston with respect to
 recoiling parts
 w - Total weight of the recoiling parts.
 w_q - Equivalent weight of recoiling parts.

I. Introduction

The recoil mechanism is designed to minimize the effects of structural fatigue and instability on a gun carriage by reducing and redistributing the rod pull force transmitted to the carriage of a weapon system. Nerdahl, M.C., and Frantz, J.W.⁽¹⁾ have developed non-linear models of hydropneumatic recoil mechanisms with three degrees of freedom and have designed a variable area orifice to control the energy dissipation. This recoil mechanism design can achieve the desired rod pull force-time trajectory for a specific firing zone but is far from optimum for other zones. Therefore, a control system which can adapt to different firing zones is desirable. Wu, S.M. and Madiwale, A.N.⁽²⁾ have proposed a servo valve feedback control system and a procedure for design of optimal feedback gains. Kasten, R.E., Madiwale, A.N., and Wu, S.M.⁽³⁾ have suggested a classical pressure feedback control system and an iterative trial-and-error procedure for feedback gains.

The objectives of this report are:

1. To simulate the conventional hydropneumatic recoil mechanism by using Continuous System Simulation Language (CSSL).⁽⁴⁾

2. To solve optimal control problem using mathematical programming via CSSL computer program.
3. To investigate the acceleration signals of the recoil mechanisms using the zone detector algorithm in order to obtain an automatic zone detector.
4. To deal with the microprocessor based design of an optimal adaptive orifice controller.
5. To investigate a gas chamber design by filling it with foam or composite material. (5,6)

Based on these concepts, the performance of the optimal control recoil mechanism in relation to rod pull force is evaluated for different zones for a M-37, 105 mm recoil mechanism.

II. The Model Building of Conventional Hydropneumatic Recoil Mechanism Based on the Physical Understanding of the Subsystems

The conventional hydropneumactical recoil mechanism which has been discussed in this chapter, M-37 105 mm system, is mounted on the carriage through a rigid link called the recoil rod. The carriage rests on a rigid support. The recoil mechanism shown in Fig. 1, could be separated into three subsystems as shown in Figs. 2 and 3; i.e., the recoiling mass, floating piston and control rod assembly, and the recoil rod.

The functioning of the recoil mechanism is described as follows. If a breech force is applied to a recoiling mass and sets it in motion, oil is forced from the recoil cylinder to the recuperator. The major portion of the oil flows through the variable control orifice (a_3) to the pressure chamber directly behind the floating piston. Some oil flows through the orifice (a_2) to the regulator directly behind the control rod. The gas is then compressed and the gas pressure increases from its initial value. When the oil flows through the different orifice areas (a_1 , a_2 and a_3), there is a resultant force which retards recoil mechanism motion and a drop in pressure due to both the increase in velocity and to the loss of energy at sudden changes in cross section (3).

A mathematical model for hydropneumatic recoil

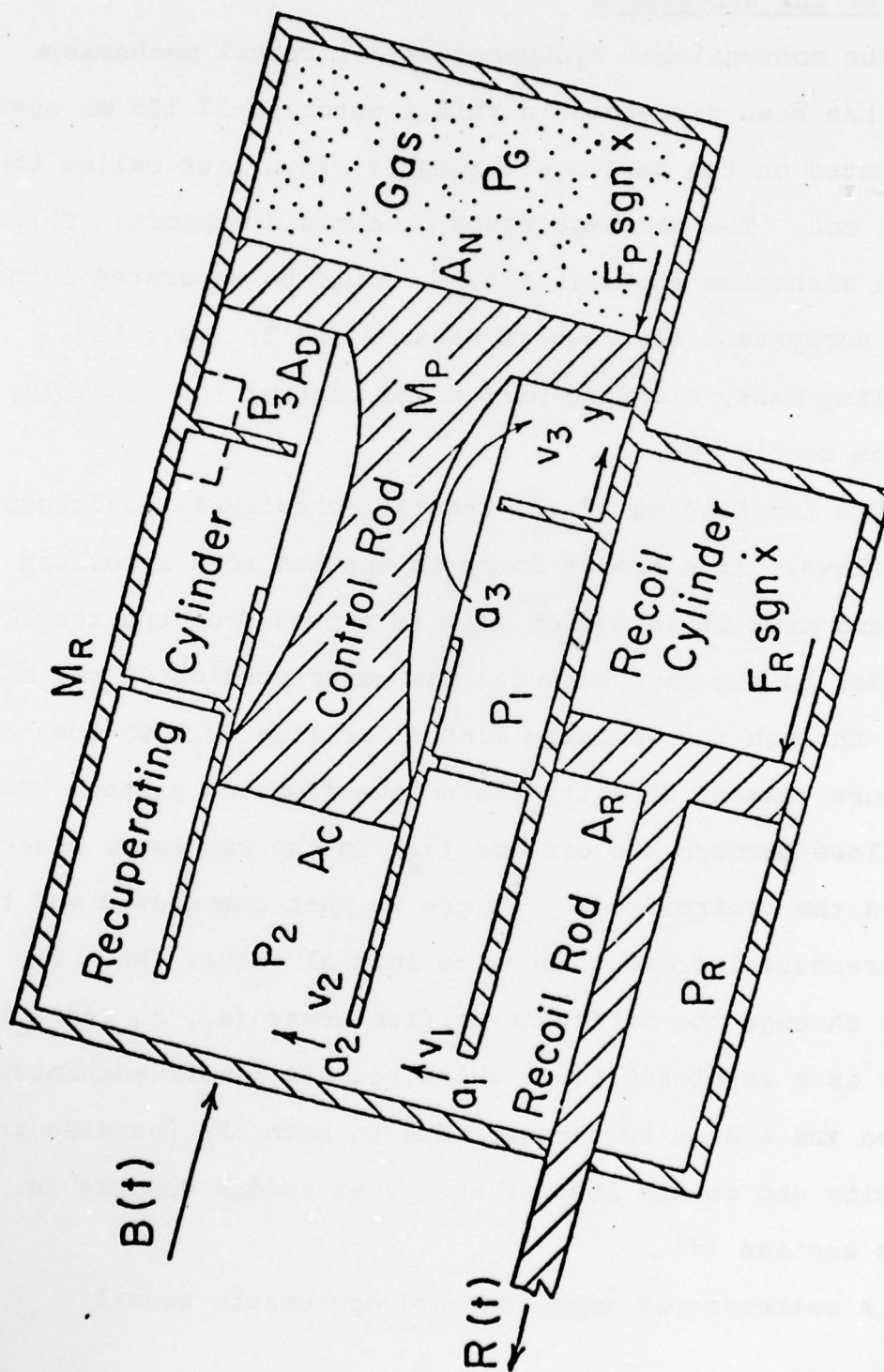


Figure 1 BLOCK DIAGRAM OF CONVENTIONAL RECOIL MECHANISM

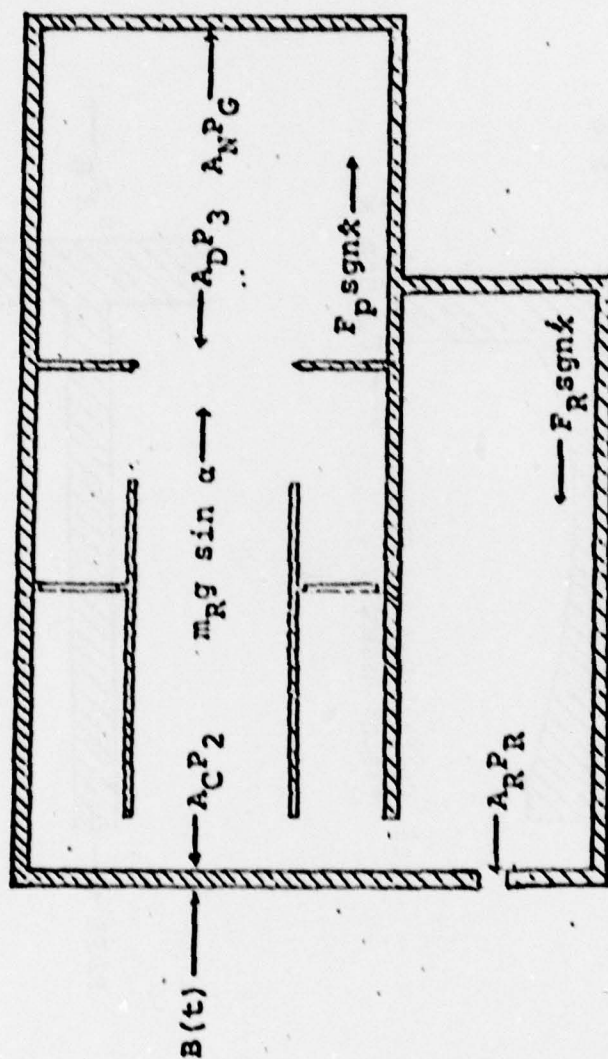


Figure 2 Free Body Diagram of Recoiling Mass

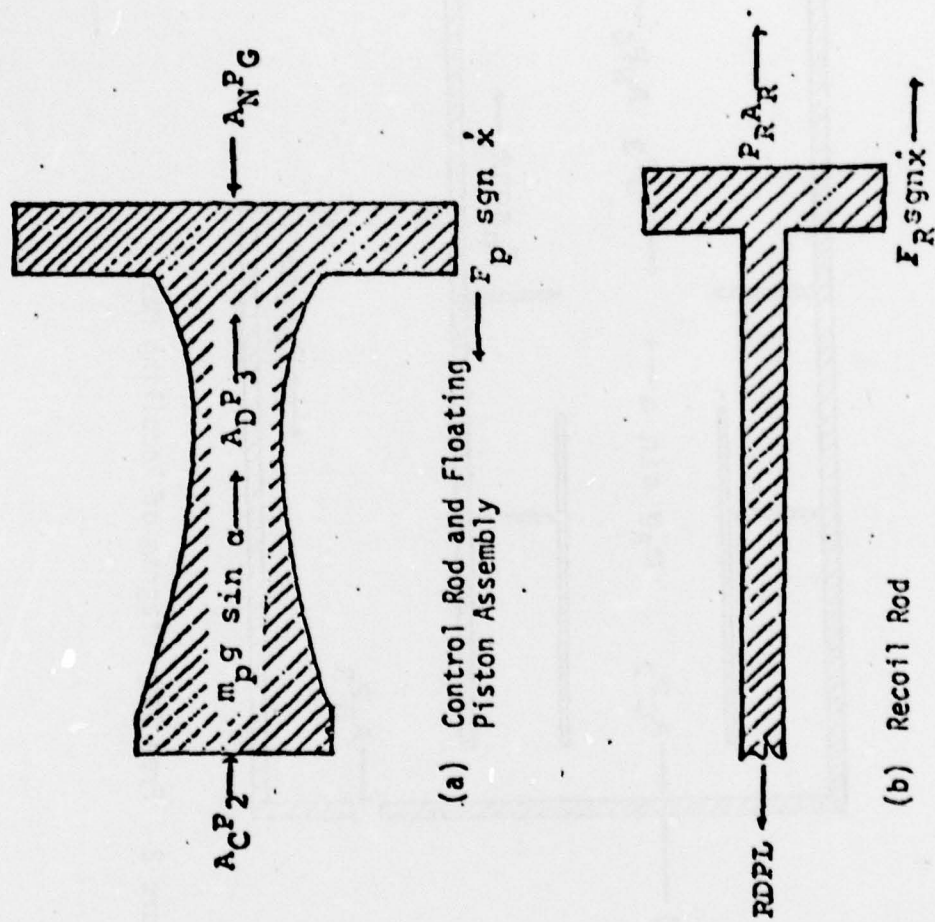


Figure 3 Free Body Diagrams of Subsystems

mechanism is based on the following assumptions:

1. The fluid is incompressible.
2. The discharge coefficient and packing friction are not affected by temperature and velocity variation.
3. Adiabatic gas law applies.
4. Only the translation of the recoil mechanism in the direction of firing is considered.
5. The system remains completely filled with oil throughout the cycle.

The equation of motion for recoil mass and floating piston follow:

$$M_R \ddot{x} = B(t) + M_R g \sin \alpha + A_N P_G + F_p \operatorname{sgn} \dot{x} - A_D P_3 - A_C P_2 - A_R P_R - F_R \operatorname{sgn} \dot{x} \quad (1)$$

$$M_p \ddot{y} = M_p g \sin \alpha + A_D P_3 + A_C P_2 - A_N P_G - F_p \operatorname{sgn} \dot{x} \quad (2)$$

Assuming that the oil volume is always completely full:

$$A_R x = A_C (y-x) + A_D (y-x) \quad (3)$$

$$y = \left(1 + \frac{A_R}{A_N}\right) x \quad (4)$$

$$\dot{y} = \left(1 + \frac{A_R}{A_N}\right) \dot{x} \quad (5)$$

$$\ddot{y} = \left(1 + \frac{A_R}{A_N}\right) \ddot{x} \quad (6)$$

The adiabatic gas pressure law is given by

$$P_G = P_O \left(\frac{V_O}{V_G}\right)^R \quad (7)$$

$$V_g = V_o - A_N(y-x) = V_o - A_R x \quad (8)$$

From the equation of continuity we have:

$$V_1 = \frac{A_R}{A_1} \dot{x} \quad (9)$$

$$V_2 = \frac{A_C}{A_2} (\dot{y} - \dot{x}) \quad (10)$$

$$V_3 = \frac{A_D}{A_3} (\dot{y} - \dot{x}) \quad (11)$$

By defining the pressure drop across the i th orifice as

$$D(V_i) = \left(\frac{f}{2}\right) \left(\frac{V_i}{C_i}\right)^2 \operatorname{sgn}(V_i), \quad (12)$$

the following pressure relations may be written

$$\begin{aligned} P_R - P_1 &= D(V_1) = \left(\frac{f}{2}\right) \left(\frac{A_R}{A_1 C_1}\right)^2 \dot{x}^2 \operatorname{sgn} \dot{x} \\ &= T_1 \dot{x}^2 \operatorname{sgn} \dot{x} \end{aligned} \quad (13)$$

$$P_1 - P_2 = \left(\frac{f}{2}\right) \left(\frac{A_C A_R}{C_2 A_2 A_N}\right)^2 \dot{x}^2 \operatorname{sgn} \dot{x} = T_2 \dot{x}^2 \operatorname{sgn} \dot{x} \quad (14)$$

$$P_1 - P_3 = \left(\frac{f}{2}\right) \left(\frac{A_D A_R}{C_3 A_3 A_N}\right)^2 \dot{x}^2 \operatorname{sgn} \dot{x} = T_3 \dot{x}^2 \operatorname{sgn} \dot{x} \quad (15)$$

By substituting for P_2 , P_3 , P_R , and P_G in Equations (7), (13), (14) and (15) and then substituting in Eq. (2)

and regrouping terms, we get

$$\begin{aligned} P_1 &= m_p \left(\frac{1}{A_N}\right) \left(1 + \frac{A_R}{A_N}\right) \ddot{x} + P_o \left(\frac{V_o}{V_o - A_R x}\right)^R + \left(\frac{1}{A_N}\right) F_p \operatorname{sgn} \dot{x} \\ &\quad - \left(\frac{1}{A_N}\right) m_p g \sin \alpha - (A_D T_3 + A_C T_2) \dot{x}^2 \operatorname{sgn} \dot{x}. \end{aligned} \quad (16)$$

Summing Equations (1), (2) and (16) we have

$$\begin{aligned}
 [m_R + m_P (1 + \frac{A_R}{A_N})] \ddot{x} &= (m_P + m_R) g \sin \alpha + B(t) \\
 -F_R \operatorname{sgn} \dot{x} - A_R T_1 \dot{x}^2 \operatorname{sgn} \dot{x} - \frac{A_R}{A_N} [m_P (1 + \frac{A_R}{A_N}) x \\
 - m_P g \sin \alpha + (A_C T_2 + A_D T_3) \dot{x}^2 \operatorname{sgn} \dot{x} + A_N P_O (\frac{V_O}{V_O - A_R x})^R \\
 + F_P \operatorname{sgn} \dot{x}] \\
 &= [m_R + m_P (1 + \frac{A_R}{A_N})] g \sin \alpha + B(t) - (F_R + F_P \frac{A_R}{A_N}) \operatorname{sgn} \dot{x} \\
 &- [A_R T_1 + \frac{A_R}{A_N} (A_C T_2 + A_D T_3)] \dot{x}^2 \operatorname{sgn} \dot{x} - A_R P_O (\frac{V_O}{V_O - A_R x})^R \\
 &- \frac{A_R}{A_N} m_P (1 + \frac{A_R}{A_N}) \ddot{x} .
 \end{aligned}$$

The governing equations of motion may be written as

$$\begin{aligned}
 [m_R + m_P (1 + \frac{A_R}{A_N})^2] \ddot{x} + [A_R T_1 + \frac{A_R}{A_N} (A_C T_2 + A_D T_3)] \dot{x}^2 \operatorname{sgn} \dot{x} \\
 + A_R P_O (\frac{V_O}{V_O - A_R x})^R + (F_R + F_P \frac{A_R}{A_N}) \operatorname{sgn} \dot{x} \\
 = m_R + m_P (1 + \frac{A_R}{A_N}) g \sin \alpha + B(t) .
 \end{aligned} \tag{17}$$

By defining the following terms:

$$m_q = m_R + m_P (1 + \frac{A_R}{A_N})^2$$

$$C_q = A_R T_1 + \frac{A_R}{A_N} (A_C T_2 + A_D T_3)$$

$$\dot{F}_q = F_R + F_P \frac{A_R}{A_N}$$

$$W_q = [m_R + m_p(1 + \frac{A_R}{A_N})] g \sin \alpha$$

equation (16) may be rewritten:

$$m_q \ddot{x} + C_q \dot{x}^2 \operatorname{sgn} \dot{x} + F_q \operatorname{sgn} \dot{x} + A_R P_O \left(\frac{1}{1 - \frac{A_R}{V_O} x} \right)^R = W_q + B(t) \quad (18)$$

For rod pull force we have:

$$RDPL = P_R A_R + F_R \operatorname{sgn} \dot{x} \quad (19)$$

From Equations (1) and (18) we get

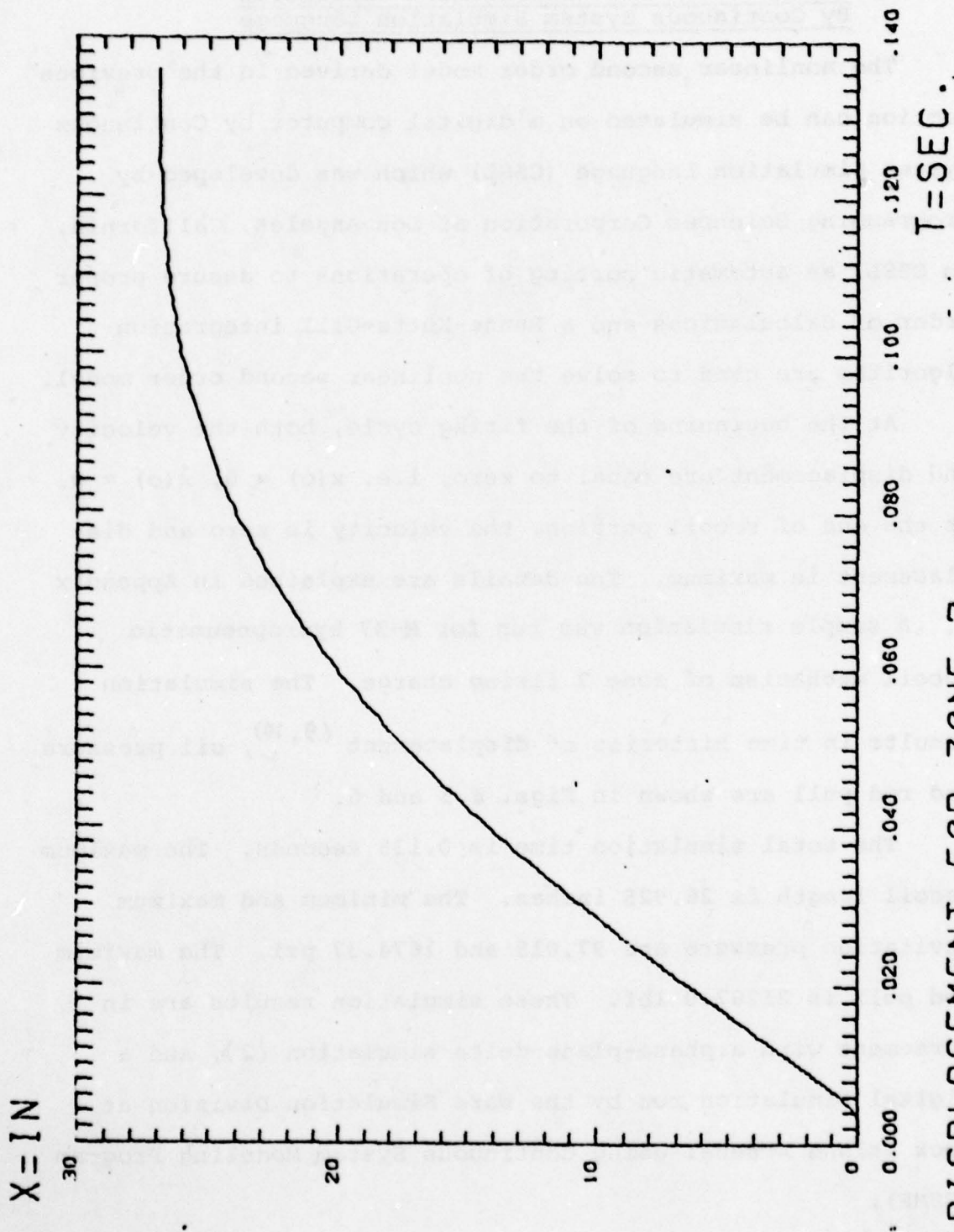
$$RDPL = -[m_R + m_p(1 + \frac{A_R}{A_N})] \ddot{x} + (m_p + m_R) g \sin \alpha + B(t) \quad (20)$$

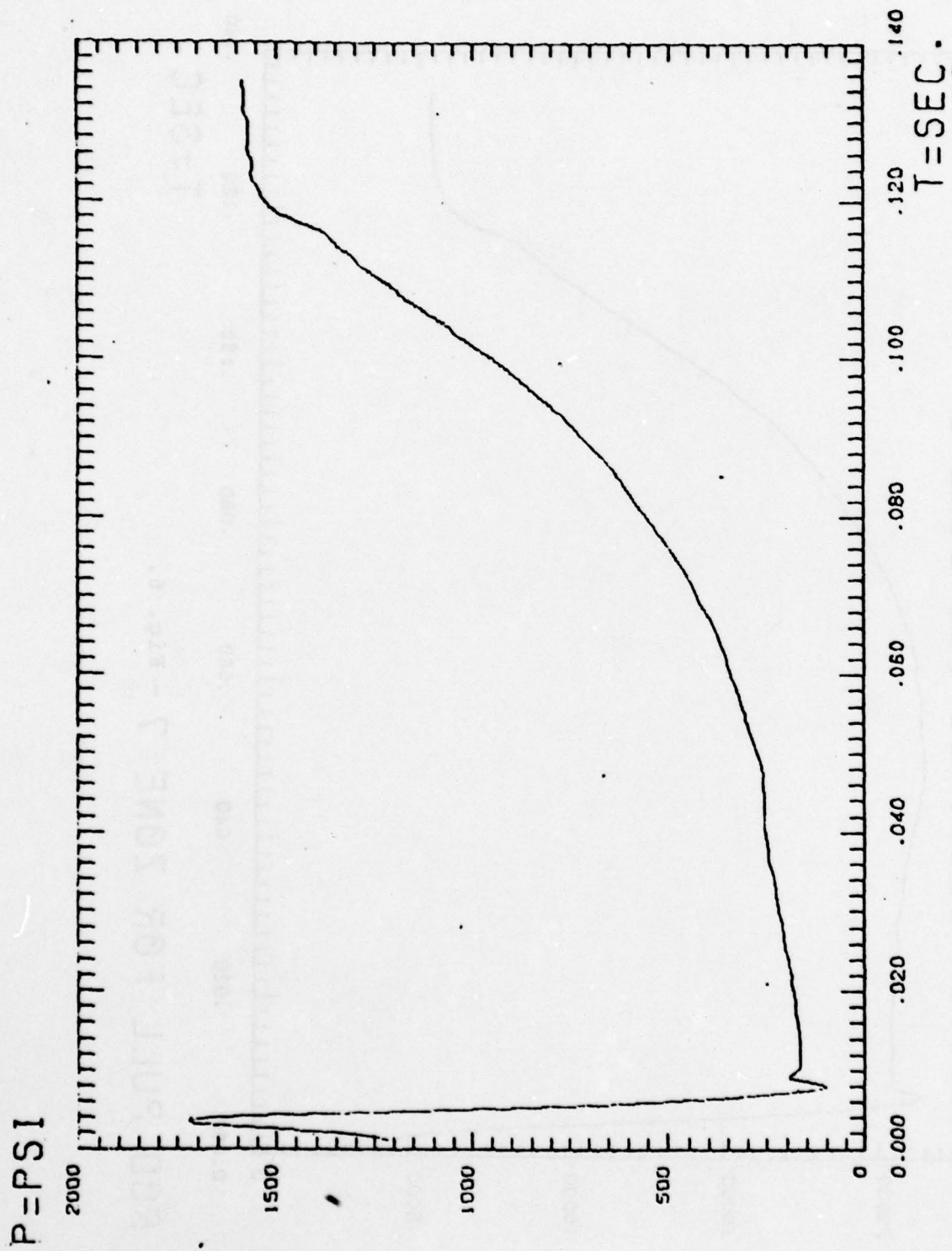
III. Conventional Recoil Mechanism Simulation By Continuous System Simulation Language

The nonlinear second order model derived in the previous section can be simulated on a digital computer by Continuous System Simulation Language (CSSL) which was developed by Programming Sciences Corporation at Los Angeles, California. In CSSL, an automatic sorting of operations to assure proper order of calculations and a Runge-Kutta-Gill integration algorithm are used to solve the nonlinear second order model.

At the beginning of the firing cycle, both the velocity and displacement are equal to zero, i.e. $x(0) = 0$, $\dot{x}(0) = 0$. At the end of recoil portion, the velocity is zero and displacement is maximum. The details are explained in Appendix I. A sample simulation was run for M-37 hydropneumatic recoil mechanism of zone 7 firing charge. The simulation results in time histories of displacement (9,10), oil pressure and rod pull are shown in Figs. 4,5 and 6.

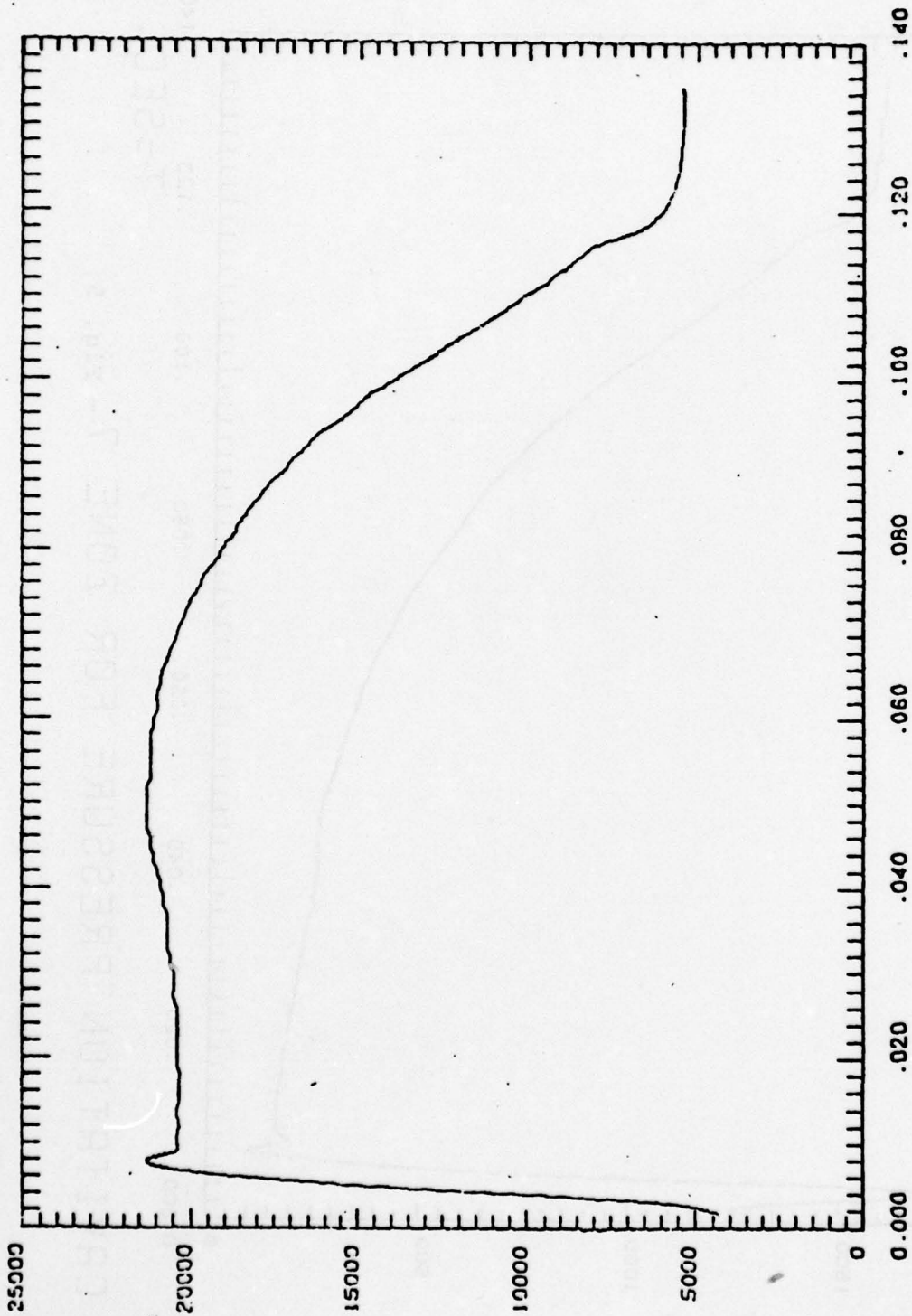
The total simulation time is 0.135 seconds. The maximum recoil length is 26.925 inches. The minimum and maximum cavitation pressure are 97.018 and 1674.37 psi. The maximum rod pull is 21287.0 lbf. These simulation results are in agreement with a phase-plane-delta simulation (2), and a digital simulation run by the Ware Simulation Division at Rock Island Arsenal using Continuous System Modeling Program (CSMP).





CAVITATION PRESSURE FOR ZONE 7 - Fig. 5.

ROPL=LBF



ROD PULL FOR ZONE 7 - Fig. 6.

IV. Optimal Control by Mathematical Programming

4.1 Modified Recoil Mechanism

The original system follows the model:

$$m_q \ddot{x} + C_q \dot{x}^2 \operatorname{sgn} \dot{x} + F_q \operatorname{sgn} \dot{x} + A_R P_O (1 - \frac{A_R}{V_O} x)^{-R} = W_q + B(t) \quad (21)$$

where

$$C_q = A_R T_1 + \frac{A_R}{A_N} (A_C T_2 + A_D T_3) \quad (22)$$

$$T_3 = \left(\frac{g}{2}\right) \left(\frac{A_D}{C_3} \frac{A_R}{A_N}\right)^2 \frac{1}{(A_3 + u)^2}$$

u is the variable orifice area.

There are six types of optimal orifice control laws for the variable orifice area which are investigated:

1. $u = A_3(x)$ where $A_3(x)$ is the area carved in the floating piston.
2. $u = q_1 x + q_2 \dot{x}$ where q_1 and q_2 are optimum values of gains.
3. $u = q_1 x + q_2 \dot{x}^2$
4. $u = q_1 (5.0) + q_2 \dot{x}^2$
5. Using the oil pressure as the feedback $u = q_1 P_R(t)$, where $P_R(t)$ is the oil pressure at recoil piston.
6. The orifice area dynamics is represented by a first order differential equation with time constant τ :

$$\tau \dot{u} + u = q_1 P_R(t)$$

The block diagram of modified recoil mechanism is shown in Figure 7.

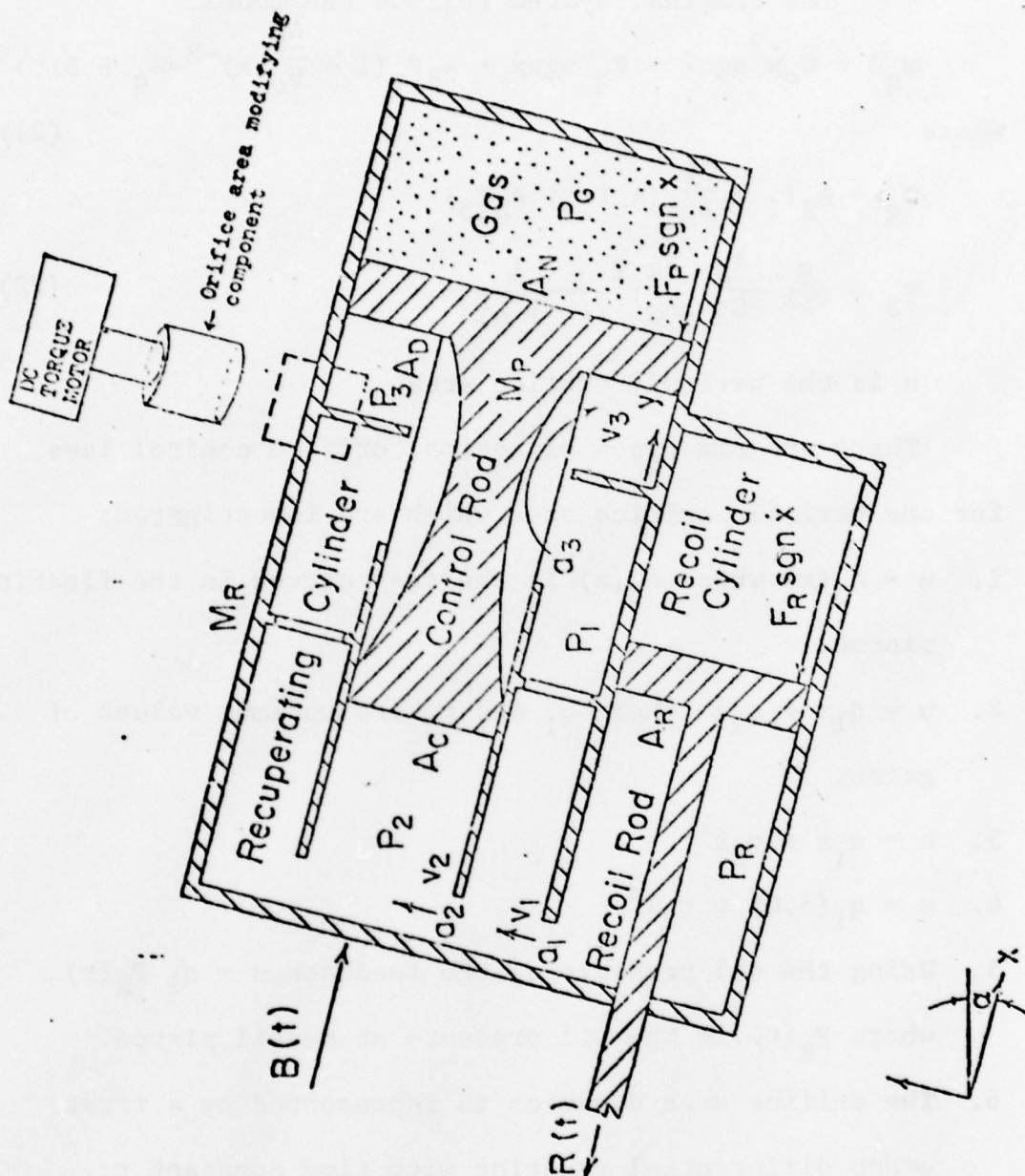


Figure 7 BLOCK DIAGRAM OF CONVENTIONAL RECOIL MECHANISM AND ORIFICE CONTROLLER SYSTEM

4.2 Formulation of Objective Function

The objective function (11,12) is formulated as an unconstrained optimization problem by two parts. The first part is the main objective of minimization of recoil force

$$J_1 = \max [R(t)], \text{ for } 0 \leq t \leq T, T \text{ is recoil time}$$

The second part will be penalty function for constraint violation and it will be active only when a constraint is violated.

The physical constraints are:

1. The maximum recoil lengths available is 28.2352 inches.
2. The oil fluid pressure $P_3(t)$ at any time is greater than zero to avoid cavitation in chamber 3.
3. The maximum available servo-valve area is 0.5 inches square.

A quadratic composite penalty function for all three constraint violations is selected as

$$J_2 = W_1 (x(t) - x_{\max})^2 + W_2 (u(t) - u_{\max})^2 + W_3 (P(t) - 0.0)^2, 0 \leq t \leq T$$

where W_1 , W_2 and W_3 are positive weighting coefficients and are set zero except when constraints are violated. Whenever a particular constraint is violated, the corresponding weighting coefficient is set to a large positive number to force the solution in the feasible region.

The composite objective function has been devised :

$$J = J_1 + \int_0^T J_2 dt$$

$$= \max [R(t)] + \int_0^T \{ W_1 (x(t) - x_{\max})^2 + W_2 (u(t) - u_{\max})^2 + W_3 (P(t) - 0.0)^2 \} dt$$

for $0 \leq t \leq T$.

4.3 Construction of the CSSL Computer Program for Modified Recoil Mechanism

The corresponding CSSL representation statements of equations (21), (22) are shown as follows:

```
SU=G1*X+G2*XDOT
BUDOT=(SU-BU1)/TOU
BU=INTEG(BUDOT,BU0)
DUMBL=WQ/MQ*B(T)/MQ-CQ*SWIN(XDOT,-1.0,1.0)*(XDOT**2)/MQ
X2DOT=DUMBL-FQ*SWIN(XDOT,-1.0,1.0)/MQ-AR*PO/((1.0-AR*X/VO)**R)/MQ
XDOT=INTEG(X2DOT,XD0)
X=INTEG(XDOT,X0)
```

The Runge-Kutta-Gill method ^(13,14) which has the form $y_{i+1} = y_i + hf'(x_i, y_i, h)$ is used as the integration algorithm. It develops one-step procedure which involves only first-order derivative evaluations, but which also produces results equivalent in accuracy to the high-order Taylor's expansion of the solution $y(x_i+h)$ about some starting points x_i :

$$y(x_i+h) = y(x_i) + hf'(x_i, y(x_i)) + \frac{h^2 f''(x_i, y(x_i))}{2!} + \frac{h^3 f'''(x_i, y(x_i))}{3!} + \dots$$

The Runge-Kutta-Gill method is stable, i.e., small errors are not amplified, and it is self-starting which means we only need the initial condition for continuous calculations. But it has disadvantages. Four separate calculations of $f(x, y)$ are required at each step. The errors of order h^5 per step are not known. We use steps of one-half the original increment h , and repeat the calculation. If the result agrees with the first or relative error 0.0001 range, then the original increment was small enough.

In CSSL, the integration algorithms utilize two intervals in performing integrations. The first is called the communication interval which is subdivided, by the integration routine, into small divisions called the calculation intervals. A calculation interval is that at which the state variables are integrated and updated. At the end of each communication interval the user can reference, or

output the values of his problem variables.

Based on the physical understanding and trial and error of determining the integration increment, the communication interval, 0.001 seconds, and the calculation interval, 6, were chosen for use in the CSSL computer program. Then, CSSL automatic sorting of operations automatically sorted the statements to achieve a proper order of calculations.

CSSL also allowed parameter alternation providing program control at run-time during which the different servo valve time constants were inputted. The function generation syntax were used for the breech force $B(t)$ and variable orifice area $A_s(x)$.

An unconstrained nonlinear optimization algorithm⁽¹⁵⁾ which is a compromise between three different algorithms for minimizing the objective function: Newton-Raphson, Steepest Descent and Marquart. It is used to solve the optimal pressure feedback gain $u(t) = q_1 P_R(t)$ problem.

The quasi-Newton method⁽¹⁵⁾ in which derivatives are estimated by differences and the second derivatives matrix is updated using the Davidon-Fletcher-Powell modification rule is used to calculate optimum values of gains q_1 and q_2 . This is found to be suitable for the $u = q_1 x + q_2 \dot{x}$ problem.

CSSL computer programs were developed in conjunction with the derivation of the mathematical models and optimization algorithm to study the effect of various orifice control laws on the optimal control for the M-37 recoil mechanism listed in Appendix I.

4.4 Results for the M-37 Recoil Mechanism

The design data and breech forces for zone 5, 6, 7 and 8 are tabulated in Appendix II and are provided by the Ware Simulation Division at Rock Island Arsenal.

The effect of orifice area equation $u = A_3(x)$ for zones 5, 6, 7 and 8 on the performance of the M-37 recoil system in relation to rod pull force are presented in Table 1.

Table 1. Results of $u = A_3(x)$ for Rod Pull Force from Zones 5 and 8.

Zone	Recoil Length (in)	Recoil Time (sec.)	Max. Rod Pull Force (lbs.)	Max. Orifice Area (in. ²)
8	27.0254	0.1219	24,382	0.0
7	26.8978	0.1329	21,287	0.0
6	26.0102	0.1679	14,681	0.0
5	24.0360	0.1889	11,166	0.0

The effect of optimal orifice control law $u = q_1x + q_2\dot{x}$, $u = q_1x + q_2\dot{x}^2$, $u = q_1(5.0) + q_2\dot{x}^2$, $u = q_1 P_R(t)$ and $\tau\dot{u} + u = q_1 P_R(t)$ for zones 5, 6, 7 and 8 on the performance of the M-37 recoil system with the maximum rod pull force are presented in Tables 2, 3, 4, 5 and 6.

Table 2. Results of $u = q_1x + q_2\dot{x}$ for Rod Pull Force from zones 5 and 8.

Zone	Gains		Recoil Length (in)	Recoil Time (sec.)	Max. Rod Pull Force (lbs.)	Max. Orifice Area (in. ²)
	q_1	q_2				
8	3.1321×10^{-4}	8.9021×10^{-8}	27.9049	0.1249	24,155	0.1044
7	3.7114×10^{-4}	5.3478×10^{-6}	28.0513	0.1349	20,406	0.1069
6	3.0537×10^{-4}	3.8326×10^{-5}	27.6529	0.1589	12,533	0.1173
5	5.8380×10^{-4}	1.0598×10^{-4}	28.0689	0.1939	8,510	0.1349

Table 3. Results of $u = q_1x + q_2\dot{x}^2$ for Rod Pull Force from Zones 5 and 8.

Zone	Gains		Recoil Length (in.)	Recoil Time (sec.)	Max. Rod Pull Force (lbs.)	Max. Orifice Area (in. ²)
	q_1	q_2				
8	3.0162×10^{-4}	1.4727×10^{-10}	27.8618	0.1249	24,176	0.1043
7	3.3682×10^{-4}	1.5257×10^{-8}	27.8848	0.1329	20,310	0.1072
6	5.6228×10^{-4}	1.0610×10^{-7}	28.1470	0.1659	12,640	0.1179
5	7.8367×10^{-4}	3.6693×10^{-7}	27.8636	0.1959	8,652	0.1345

Table 4. Results of $u = q_1(5.0) + q_2\dot{x}^2$ for Rod Pull Force from Zones 5 and 8.

Zone	Gains q_1 q_2	Recoil Length (in.)	Recoil Time (sec.)	Max. Rod Pull Force (lbs.)	Max. Orifice Area (in. ²)
8	2.1824×10^{-3} -4.4151×10^{-8}	28.1802	0.1289	23,786	0.1041
7	2.1449×10^{-3} -3.3795×10^{-8}	28.0493	0.1349	20,142	0.1074
6	2.8402×10^{-4} 5.2730×10^{-8}	26.5072	0.1639	13,434	0.1103
5	4.7222×10^{-3} 6.3040×10^{-8}	27.8398	0.1969	8,569	0.1313

Table 5. Results of $u = q_1 P_R(t)$ for Rod Pull Force from Zones 5 and 8.

Zone	Gain q_1	Recoil Length (in.)	Recoil Time (sec.)	Max. Rod Pull Force (lbs.)	Max. Orifice Area (in. ²)
8	4.0×10^{-7}	27.2725	0.1159	25,539	0.1057
7	9.8×10^{-7}	27.4926	0.1249	20,990	0.1088
6	4.1675×10^{-6}	28.1157	0.1619	12,424	0.1189
5	1.13×10^{-5}	28.2055	0.1979	8,500	0.1325

Table 6. Results of $\tau\dot{u} + u = q_1 P_R(t)$ for Rod Pull Force from Zone 5 and 8.

Zone	Gain q_1	Recoil Length (in.)	Recoil Time (sec.)	Max. Rod Pull Force (lbs.)	Max. Orifice Area (in. ²)
8	6.2×10^{-7}	27.4713	0.1169	25,351	0.1065
7	1.12×10^{-7}	26.9685	0.1319	21,242	0.1034
6	4.275×10^{-6}	28.1716	0.1639	12,897	0.1182
5	1.12×10^{-5}	28.1183	0.1989	8,980	0.1315

Comparing tables 1, 2, 3, 4, 5 and 6 of the maximum rod pull force, it can be seen that the last choice of optimal control law for zones 5 and 6 is $u = q_1 P_R(t)$, and for zones 7 and 8 is $u = q_1(5.0) + q_2 \dot{x}^2$.

The values of the optimal control laws (u) and rod pull force with different time (t) for zones 5, 6, 7 and 8 are tabulated in Appendix IV.

V. Design for Automatic Optimal Controller

5.1 Automatic Zone Detector

A microprocessor based hardware configuration for the zone detector is illustrated in Figure 8. This configuration of a microprocessor consists of a clock, power supply and transducers for measuring position, velocity, and recoil acceleration.

The analog signals from the transducers are transferred to the microprocessor through a multiplexer, an instrument amplifier, a sample-and-hold and an analog-to-digital converter. These digitized signals are tested by zone identification algorithms. The microprocessor then sends out the zone identification signal, appropriate for the firing zone charge measured.

There are two diagnostic tools for the purpose of automatic zone identification that are investigated: the

first tool is the acceleration peak value analysis; the second is the Dynamic Data System Method.

5-1.1 The Acceleration Peak Value Analysis

The recoil acceleration signals have been found to have distinct peak values for the different zones which are determined by using the zone detector computer program. The zone detector computer program flow chart is shown in Figure 9 and the computer program is shown in Figure 10. These peak values are shown in Table 7. As a consequence of these different peak value characteristics, the recoil acceleration can serve as a diagnostic tool for the purpose of automatic zone identification.

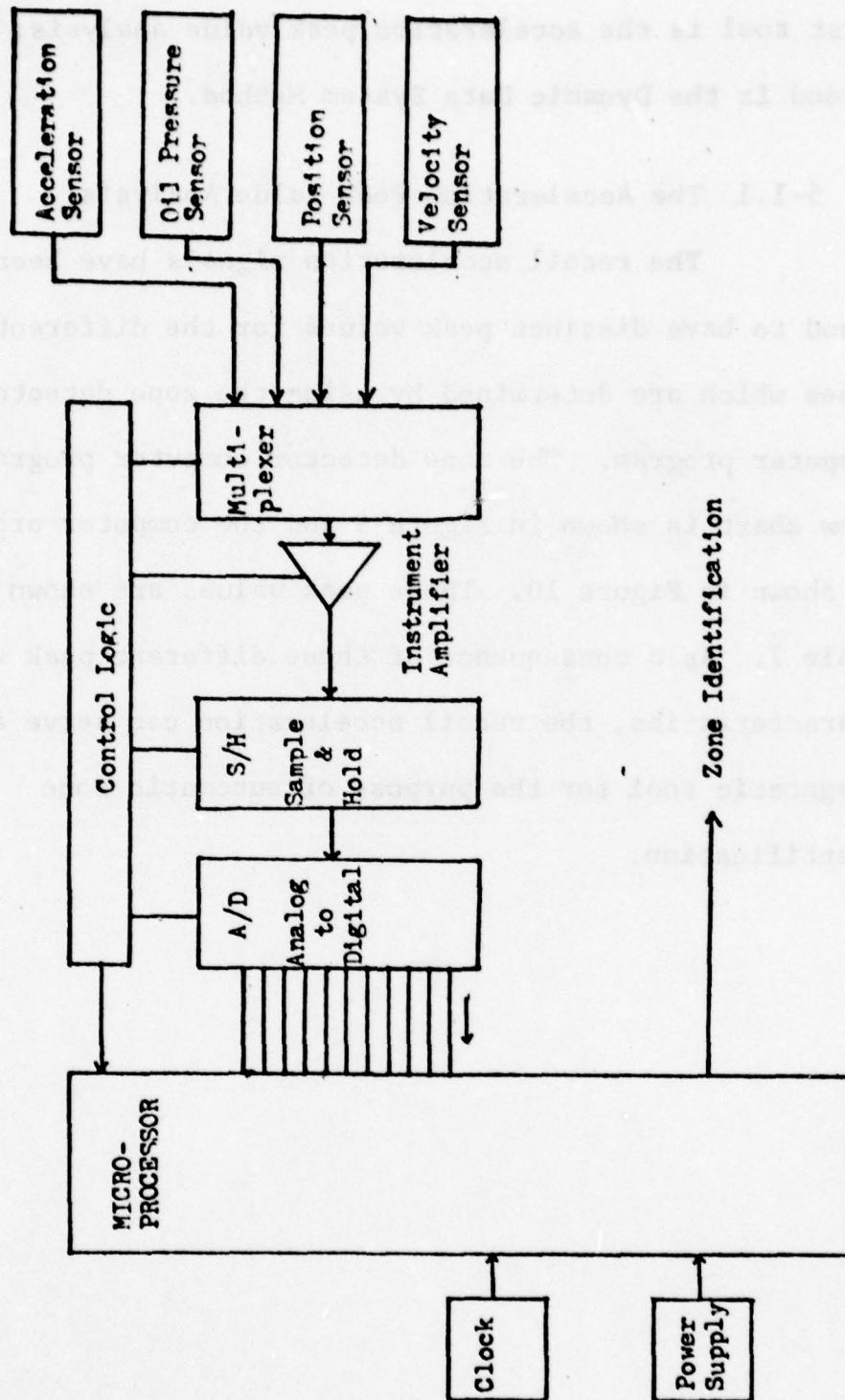


Figure 8 Hardware Configuration for Zone Detector

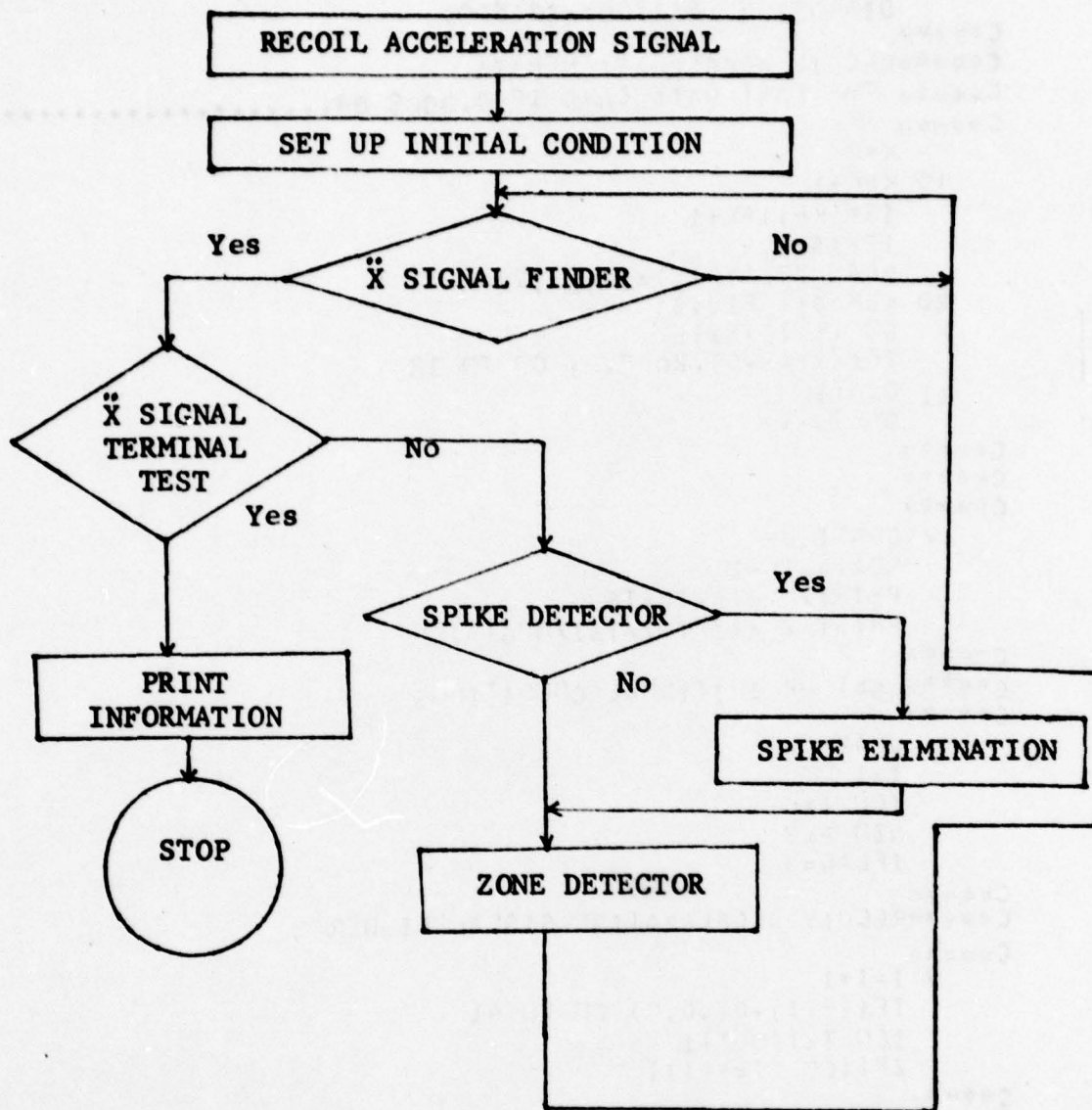


Figure 9 Flow Chart of Zone Detector Program

```

      DIMENSION YP(1800), ZP(500)
C*****
C*****RECDIL ACCELERATION DATA
C***** THE LAST DATA CARD IS 0.300E 08.....
C*****
      K=0
    10 K=K+1
      IS=(K-1)*8+1
      IE=IS+7
      READ 20, (YP(I), I=IS, IE)
    20 FORMAT(1E10.3)
      DO 11 I=IS, IE
        IF(YP(I).GT.2000.0) GO TO 12
    11 CONTINUE
      GO TO 13
C*****
C*****
C*****
    12 CONTINUE
      NDATA=IE-1
      PRINT, NDATA, IE, IS
      PRINT 2, (YP(I), I=1, NDATA)
C*****
C***** SET UP INITIAL CONDITIONS
C*****
      YMAX=0.0
      I=1
      ICDT=0
      NZONE=7
      IFLAG=0
C*****
C*****RECDIL ACCELERATION SIGNAL FINDER
C*****
    9 I=I+1
      IF(YP(I).GT.0.0) GO TO 41
      ICDT=ICDT+1
      ZP(ICDT)=YP(I)
C*****
C*****SPIKE DETECTOR AND SPIKE ELIMINATOR
C*****
      DELTA=ABS(YP(I)-YP(I-1))
      IF(DELTA.GT.90.0) GO TO 14
      GO TO 15
C*****
C*****SPIKE ELIMINATOR
C*****
    14 CONTINUE
      IF(DELTA.GT.150.0) GO TO 95
      YY=YP(I-2)
      YP(I-1)=YY
      YP(I)=YY
      YP(I+1)=YY
      YP(I+2)=YY
      YP(I+3)=YY

```

Figure 10 Zone Detector Program

```

      YP(I+4)=YY
      YP(I+5)=YY
95  ZP(ICON,T)=YP(I-2)
      GO TO 30
15  IF(YP(I).GT.YMAX) GO TO 30
      IFLAG=1
      IPC=ICON-T
      IPEAK=I
      YMAX=YP(I)
      IF(YMAX.LT.-75.0) GO TO 21
      NZONE=1
      PEAK=YMAX
      GO TO 30
21  IF(YMAX.LT.-90.0) GO TO 22
      NZONE=2
      PEAK=YMAX
      GO TO 30
22  IF(YMAX.LT.-120.0) GO TO 23
      NZONE=3
      PEAK=YMAX
      GO TO 30
23  IF(YMAX.LT.-150.0) GO TO 24
      NZONE=4
      PEAK=YMAX
      GO TO 30
24  IF(YMAX.LT.-185.0) GO TO 25
      NZONE=5
      PEAK=YMAX
      GO TO 30
25  IF(YMAX.LT.-295.0) GO TO 26
      NZONE=6
      PEAK=YMAX
      GO TO 30
26  IF(YMAX.LT.-380.0) GO TO 27
      NZONE=7
      PEAK=YMAX
      GO TO 30
27  YMAX=0.0
      NZONE=7
      PEAK=YMAX
      GO TO 30
30  CONTINUE

C*****
C***** RECOIL ACCELERATION TESTOR
C*****
      GO TO 9
41  CONTINUE
      IF(ICON.T.GT.50) GO TO 31
      ICON=T
      IFLAG=0
31  IF(IFLAG.GT.0) GO TO 50
      GO TO 9
50  PRINT 6,ZONE,I,ICON,IFLAG
      PUNCH 20,(ZP(I)),I=1,ICON
      GO TO 41

C*****
C*****
      PRINT 2,(ZP(I)),I=1,ICON

```

5-1.2 Dynamic Data System (DDS)⁽¹⁷⁾ Method

The recoil acceleration signals are shown in Appendix III. The DDS approach is used to obtain the dynamic information of the recoil acceleration signals. Time series model AR(1), AR(2) and AR(3) are fitted to the above data, and the result is shown in the Tables 8, 9 and 10. In the Tables 8, 9 and 10 we use the "entire" period of the acceleration signals to fit the models. The "entire" period is defined as the time interval between the starting point of acceleration and the point where the acceleration signal goes back to zero from negative values.

Since there are many "spikes" at the beginning and at the end of the period of the acceleration signals, the AR(1), AR(2), and AR(3) models in the tables 8, 9, and 10 do not show distinct parameter value characteristics.

A "half" period is defined as the time interval between the starting point of acceleration and the point where the signal has its peak.

Then the half period of the acceleration signal is used to fit AR(1) model which is shown in Table 11. It appears that the parameter θ and the mean have distinct and stable values for each zone. Using the "half" period

acceleration signal, AR(1) model can serve as a diagnostic tool for the purpose of automatic zone identification.

Table 7. Peak Values of Recoil Acceleration

Zone	Round	Peak Values of Recoil
		Acceleration: GS
1	23	72.4
2	27	78.7
3	34	102
	35	106
4	13	*
	14	141
5	8	160
	11	159
6	41	206
	42	217
7	19	366
	20	362

Table 8. AR(1) Model for "Whole" Period of the Acceleration Signal.

Zone	Round	AR(1)	
		ϕ_1	Mean
1	23	.9929	.1168
2	27	.9951	.1507
3	34	.9884	.1604
4	14	.9928	.2731
5	8	.9896	.4261
6	41	.9909	.1924
	42	.9908	.2671
7	19	.9763	.5399
	20	.9175	.3699

Table 9. AR(2) Model for "Whole" period of the Acceleration Signal.

Zone	Round	AR(2)				Mean
		ϕ_1	ϕ_2	λ_1	λ_2	
1	23	1.5041	-.5149	.5070	.9971	.0516
2	27	1.5738	-.5837	.5984	.9755	.0652
3	34	1.6932	-.7064	.7449	.9482	.03541
	35	1.5803	-.5948	.6182	.9621	.02171
4	14	1.3623	-.3729	.3794	.9829	.1620
5	8	1.4850	-.5014	.5191	.9657	.2663
	11	1.4274	-.4390	.4483	.9789	.1257
6	41	1.1557	-.1664	.1686	.9872	.1561
	42	1.4106	-.4216	.4299	.9807	.1688
7	19	1.1173	-.1437	.1484	.9689	.5259
	20	.5781	.3676	-.3826	.9608	.4033

Table 10. AR(3) Model for "Whole" Period of the Acceleration Signal.

Zone	Round	AR(3)						Mean
		ϕ_1	ϕ_2	ϕ_3	λ_1	λ_2	λ_3	
1	23	1.466	-.404	-.075	-.124	.618	.972	.045
2	27	1.483	-.334	-.160	-.223	.741	.965	.0592
3	34	1.699	-.712	-.001	-.001	.756	.944	.0341
	35	1.548	-.498	-.066	-.099	.698	.949	.0148
4	14	1.304	-.161	-.155	-.267	.595	.977	.1365
5	8	1.514	-.580	.0481	.115	.435	.965	.268
	11	1.476	-.596	.109	.246+ .225j	.246- .225j	.983	.141
6	41	1.143	-.077	-.076	-.211	.368	.985	.146
	42	1.398	-.382	-.028	-.059	.478	.979	.165
7	19	1.1335	-.2923	.135	.078+ .364j	.078- .364j	.9757	.5235
	20	.5238	.2778	.1512	-.2234+ .325j	-.2234- .325j	.9726	.4599

Table 11. AR(1) Model for "Half" Period of the Acceleration Signal.

Zone	Round	AR(1)	
		ϕ	Mean
1	23	.90	-.0248839
2	27	.9747	-.586362
3	34	.9806	-.950448
	35	.9864	-.902341
4	14	.9897	-1.04555
5	8	.9946	-.135395
	11	.9899	-.137575
6	41	.9932	-1.81667
	42	.9983	-1.90869
7	19	1.0004	-3.68192
	20	.9997	-3.83094

5-2. Design of a Microprocessor Based Orifice Controller.

A microprocessor based hardware configuration of the optimal adaptive orifice controller is suggested in Figure 11. It consists of a microprocessor with clock and power supply, DC torque motor, and orifice area modifying component.

The microprocessor will choose the appropriate control according to the zone identification signal which

is transferred from the zone detector. The microprocessor selects the optimal control output. It is then converted to an analog signal via a digital to analog converter. The signal is amplified and used to control a DC torque motor which controls the orifice area.

The three views of the floating piston^(1,18) are shown in Figure 12. The orifice controller system is illustrated in Figure 13. This system consists of a DC torque motor with the orifice area modifying component. The DC torque motor following the adaptive control law, turns the orifice area modifying component to control the oil flow passing through the orifice slot in the floating piston.

The three views of orifice area modifying component are shown in Figure 14. The block diagram of conventional recoil mechanism with the orifice controller system is shown in Figure 7.

The zone detector and optimal adaptive orifice controller may be combined in one microprocessor to act as an optimal adaptive feedback orifice controller and it is illustrated in Figure 15.

This feedback controller consists of a microprocessor, clock, power supply, programmable interfaces, DC torque

motor, orifice area modifying component and transducers.

The analog signals from the transducers are digitized as before and are used to choose the correct control law. The microprocessor then selects the optimal output to vary the orifice area appropriately.

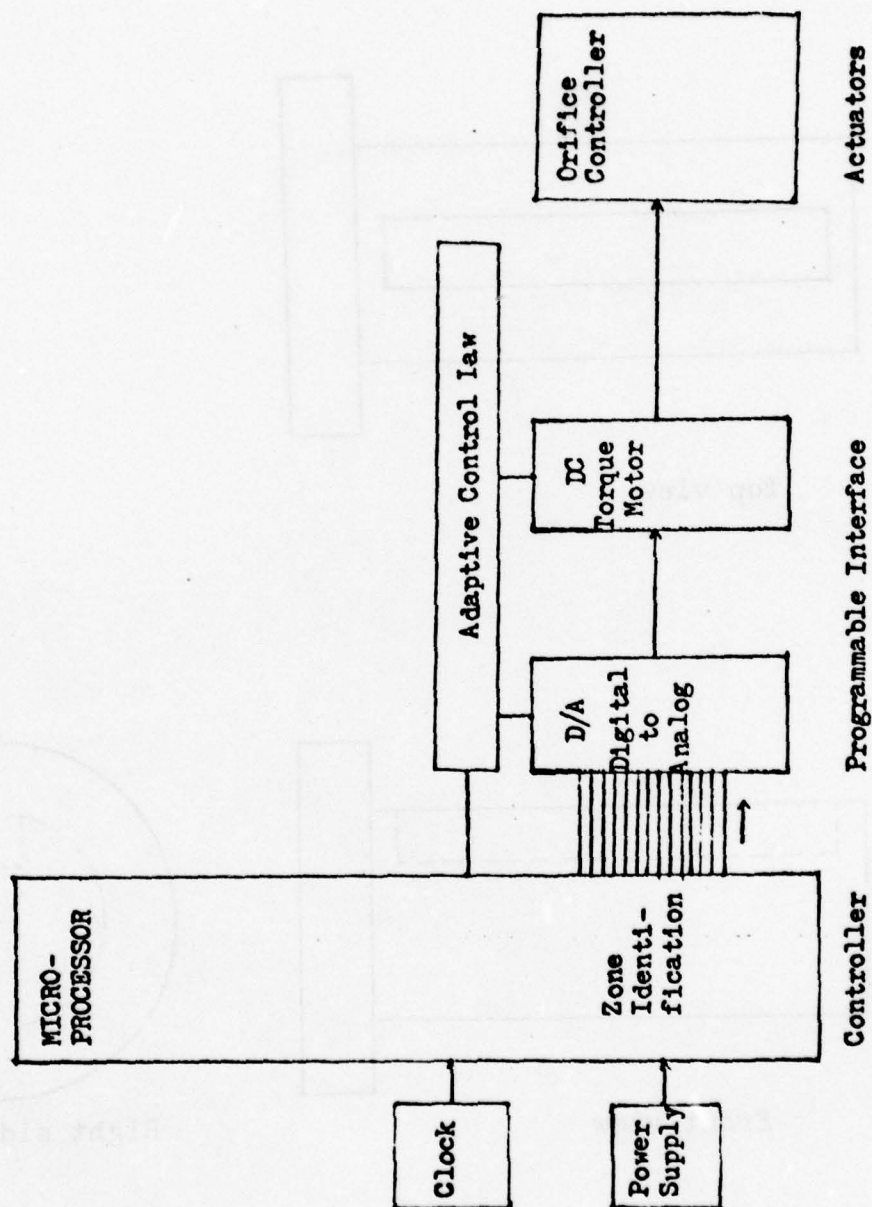
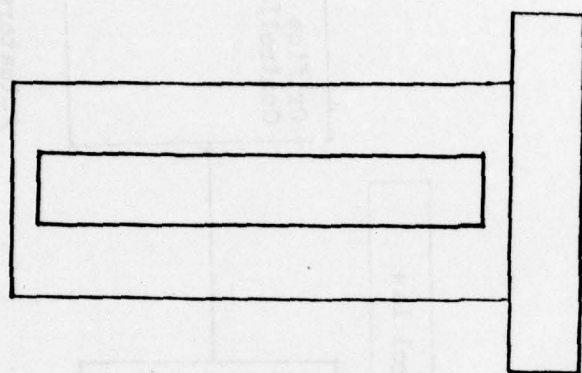
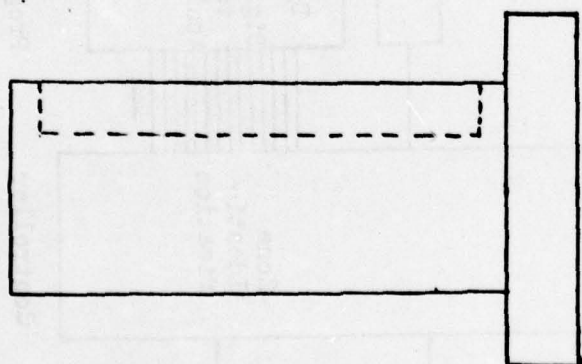


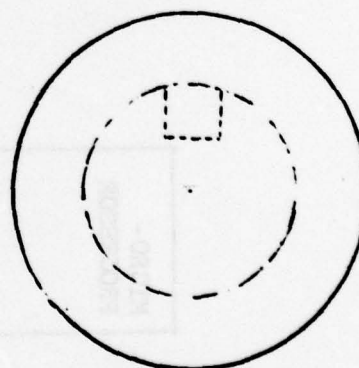
Figure 11 Hardware Configuration for Optimal Adaptive Orifice Controller



Top view



Front view



Right side view

Figure 12 The three views of floating piston

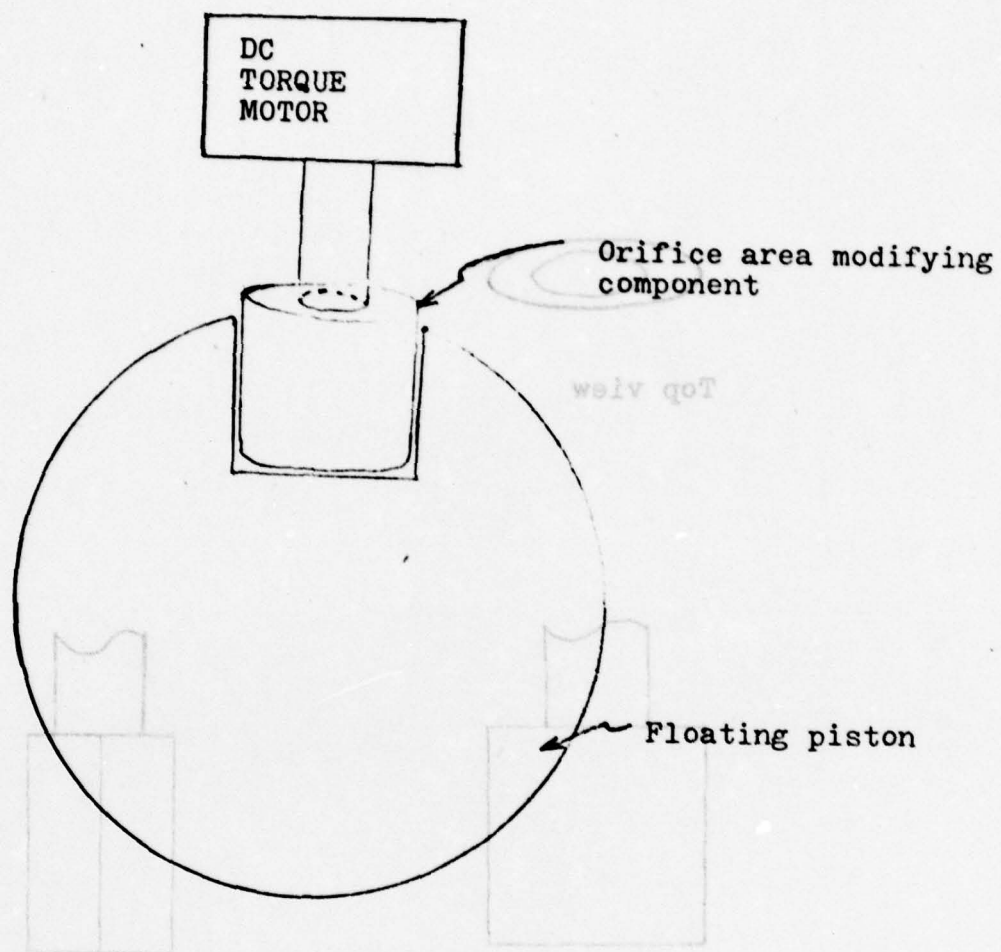
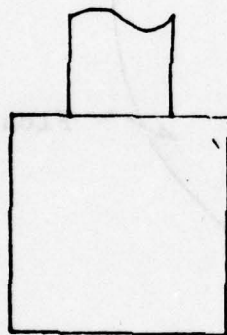


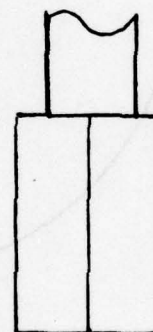
Figure 13 The orifice controller system



Top view



Front view



Right side view

Figure 14 The three views of orifice area modifying component

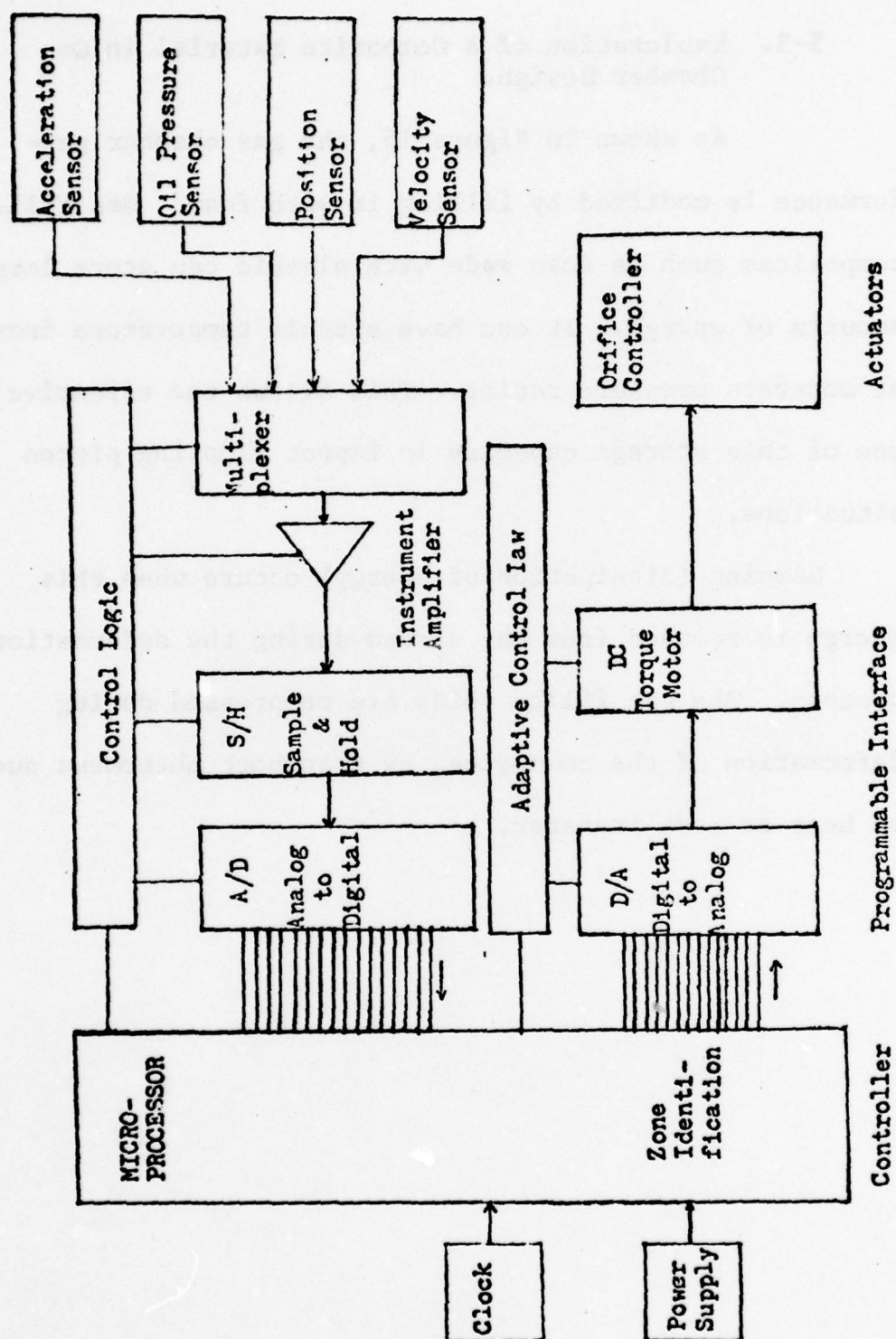


Figure 15 Hardware Configuration for Optimal Adaptive Feedback Orifice Controller

5-3. Exploration of a Composite Material in Gas Chamber Design.

As shown in Figure 16, the gas chamber performance is modified by filling it with foam. Gas filled composites such as foam made with plastic can store large amounts of energy. It can have sizable temperature increases at moderate pressure ratios. This allows the effective use of this storage capacity in impact floating piston situations.

Damping (dissipation of energy) occurs when this energy is removed from the system during the deformation process. The gas filled voids are compressed during deformation of the composite, by transport phenomena such as heat or mass transfer.

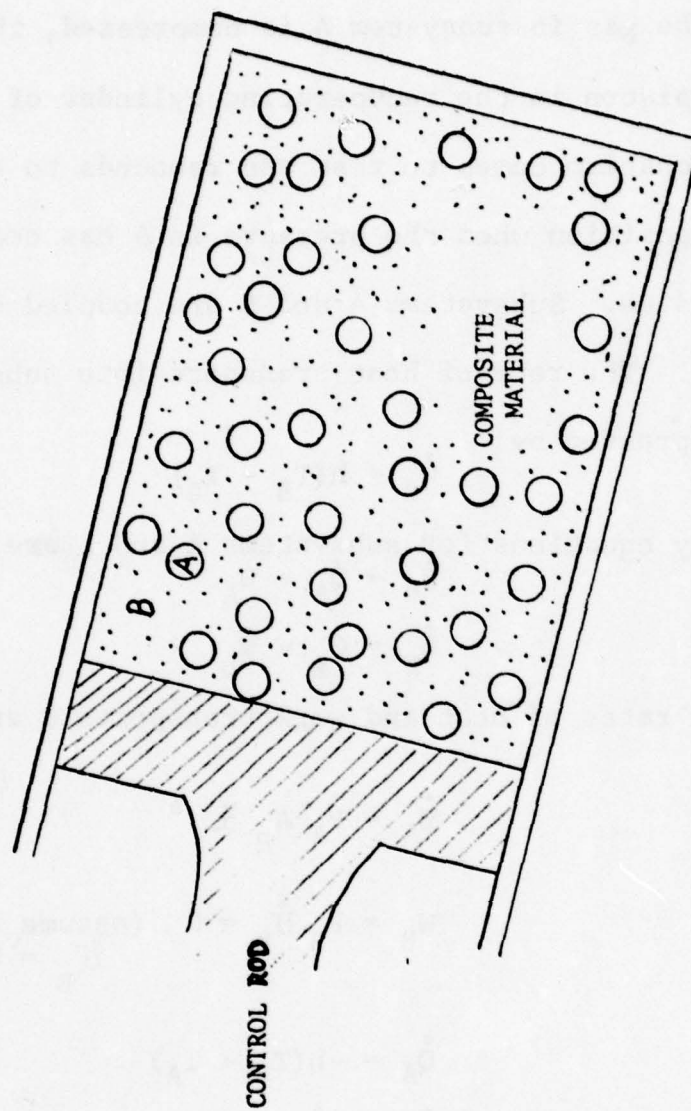


Figure 16 A Gas Filled Composite Material in Gas Chamber

The Mathematical Model of Gas-Filled Composite Materials

As the gas in subsystem A is compressed, the floating piston in the recuperating cylinder of the recoil mechanism comes to rest and rebounds to the original position when the pressure in A has reached its initial value. Subsystems A and B are coupled by heat transport. The rate of heat transport into subsystem A can be expressed by

$$\dot{Q}_A = h(T_B - T_A)$$

The energy equations for subsystems A and B are

$$\dot{U}_A = \dot{Q}_A - \dot{W}_A$$

$$\dot{U}_B = \dot{Q}_B - \dot{W}_B$$

where the rates of heat and work transports \dot{Q} and \dot{W} are given by

$$\dot{W}_A = P_A A_P \dot{x}$$

$$\dot{W}_B = P_B \dot{U}_B = 0 \quad (\text{assume incompressible...})$$

$$\dot{U}_B = 0$$

$$\dot{Q}_A = -h(T_B - T_A)$$

$$\dot{Q}_B = -\dot{Q}_A$$

System A contains an ideal gas with constant specific heat.

System B is a solid with constant specific heat. Therefore

$$\dot{U}_A = M_A C_A \dot{T}_A$$

$$P_A A_P X_{\text{eff}} = M_A R_A T_A$$

$$P_A A_P X_{\text{eff}} = P_A (A_P X_0 - M_B / \rho_B)$$

$$\dot{U}_B = M_B C_B \dot{T}_B$$

Then the temperature and pressure equation can be written in the form

$$\dot{T}_A = \frac{1}{M_A C_A} [h(T_B - T_A) - P_A A_P \dot{x}]$$

$$\dot{T}_B = \frac{1}{M_B C_B} [-h(T_B - T_A)]$$

$$P_A A_P X_{\text{eff}} = M_A R_A T_A$$

The initial conditions are

$$\left. \begin{array}{l} x = x_0 \\ \dot{x} = 0 \\ T_A = T_B = T_0 \\ P_A = P_0 \end{array} \right\} \text{ at } t=0$$

The above is the mathematical model of the gas-filled composites (foam).

A computer simulation was run for the M-37 hydro-pneumatic recoil mechanism with gas filled composites of zone 7 firing charge. The effect of different thermal conductance values from adiabatic (0.0) to isothermal (2.778×10^2) process, on the performance of the recoil system in relation to maximum rod pull forces, are presented in Table 12.

Table 12. Results of h Values for Maximum Rod Pull Forces

h Btu/sec °F	T_A °F Max. Final		T_B °F Max. Final		P_A psi	Recoil Length in.	Recoil Time sec.	Rod Pull Max. lbs.
0	103.86	103.86	77.0	77.0	1437.02	26.9457	.133999	21276.9
2.7778×10^{-1}	102.34	102.28	77.0	77.0	1433.01	26.9469	.133999	21276.9
2.7778	95.0232	91.8977	77.0065	77.0065	1406.58	26.9552	.133999	21276.8
2.7778×10^1	81.929	77.3993	77.0149	77.0149	1369.85	26.9740	.134999	21275.8
2.7778×10^2	77.583	77.0175	77.015	77.015	1368.91	26.9771	.134999	21271.4
2.7778×10^3	77.0613	77.0151	77.015	77.015	1368.91	26.9774	.134999	21268.9

Comparing the thermal conductances (h) 0.0 and 2.7778×10^3 Btu/sec. of, it can be seen that the peak force of the larger h value is about 8.0 lbf lower, the recoil length is about 0.0317 inch longer, and the recoil time is about 0.001 second longer, than those for the smaller h value. Therefore, with composite material in the gas chamber, the effect of changes in the thermal conductance on the performance of the M-37 recoil mechanism is found to be insignificant.

VI. Conclusions

1. CSSL provides a straight-forward programming tool which can easily handle complex models and makes the coding task much more simpler than an equivalent Fortran program. CSSL has proved to be a very useful tool to solve any order, linear or nonlinear, or simultaneous continuous differential equations. To carry out the detailed studies of continuous systems, CSSL is recommended as a powerful tool to gain insight into the models.

2. Based on the various orifice control laws analysis of the maximum rod pull force of zones 5, 6, 7 and 8, the optimal control greatly decreased the average rod pull peak force, and causes the force to level out to a nearly constant value following the peak. These two characteristics are desirable in the recoil mechanism design. The decreased peak force and increased stability brought about through the leveling of the rod pull force both contribute to reducing the fatigue damage to the system during operation.

3. As a consequence of different peak value characteristics, the recoil acceleration can serve as a diagnostic tool for the purpose of automatic zone detection. A microprocessor based digital controller is designed to control the orifice area controller, using the optimal

control law tables which can be stored in the read-only-memory (ROM).

4. The same microprocessor controller could be used to different recoil mechanisms by modifying the software program without changing the hardware component. The orifice controller is designed to be fail-safe. If the controller malfunctioning the recoil mechanism will still operate as before for a particular zone charge.

5. The effect of varying the thermal conductance of the composite material, on the rod pull force performance of the M-37 recoil mechanism is investigated. The major effect of changes in the thermal conductance is that the gas temperature in the gas chamber remains constant when the value of the thermal conductance approaches that of an isothermal process. Thus no significant change in the maximum rod pull force is observed.

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Appendix I

Continuous System Simulation Language (CSSL)

CSSL is a digital computer language designed to facilitate the representation and simulation of continuous dynamic systems. Extensive man-machine interactive capabilities are built into the language via graphics, plotting, parameter adjustment, etc., allowing the user to handle nonlinear and time-variant problems and enabling the user to concentrate upon the phenomenon being simulated.

To better illustrate the features of CSSL, one application has been coded in CSSL and is then followed by a brief description of what the respective statements do. Consider a system consisting of a spring, a mass and a damper suspended from a fixed reference position. If the mass is displaced from its rest position by a force BT , it will oscillate until the energy is dissipated by the damper. The purpose of a simulation might be to analyze the effect of different spring constants on the motion of the mass.

The corresponding CSSL statements are shown in the following table.

A brief description of what the respective statements do and each statement is reference by a statement number as follows (3):

CSSL Statement

```

(S1)      PROGRAM SPRING DAMPING PROBLEM
(S2)      CONSTANT XDO = 0.0, XO = 0.0, M = 3.0,
           C = 0.3, K = 1.0 BT = 1.0
(S3)      X2DOT = BT/M - C/M*XDOT - K/M*X
(S4)      XDOT = INTEG (X2DOT, XDO)
(S5)      X = INTEG (XDOT,XO)
(S6)      OUTPUT  X2DOT, XDOT,X
(S7)      TERMT (T.GE.2.0)
(S8)      END

```

Run - Time Statement

```

(S9)      START
(S10)     INPUT  C = 0.4, K = 1.2
(S11)     START
(S12)     STOP

```

- (S1) A PROGRAM card is required as the first statement of every CSSL program; a program name is optional.
- (S2) CONSTANT is used to initialize problem variables to the desired value.
- (S3) Algebraic equation which defines the variable, X2DOT.
- (S4) the INTEG operator (integrator) has the form of INTEG(F,IC) where IC is the initial value and F is the function to be integrated.

- (S5) INTEG operators may be nested to any depth.
- (S6) output will cause the values of X2DOT, XDOT, and X to be printed as the solution proceeds from zero to two seconds.
- (S7) TERMT is used to specify a terminating condition for the simulation; in this case when time is equal to or greater than two seconds.
- (S8) END signifies the end of the CSSL source statements.
- (S9) The initialization and control of problem execution and parameter alteration between runs is accomplished through the use of CSSL statements introduced at run-time as data cards. START is used to execute the problem coded (S1-S8) until the termination condition is satisfied.
- (S10) At the end of the first problem execution, the values of C and K are changed via INPUT.
- (S11) A START is again used to start problem execution.
- (S12) When the termination condition is again satisfied, STOP is used to signal the end of the total run.

CSSL previously discussed is used to study forced vibrations of a linear system. With a changing of the statement S3, CSSL can be used for forced non-linear vibration as follows:

$$\text{DUMBL} = \text{WQ}/\text{MQ} + \text{B}(\text{T})/\text{MQ} - \text{CQ} * \text{SWIN}(\text{XDOT}, -1.0, 1.0) \\ * (\text{XDOT} ** 2) / \text{MQ}$$

$$X2DOT = DUMBL - FQ*SWIN(XDOT, -1.0, 1.0) / MQ - AR*PO /$$
$$((1.0 - AR*X/VO)**R) / MQ$$

After we add the following statements to the above representation statements

$$SU = G1*X + G2*XDOT$$

$$BUDOT = (SU - BU) / TOU$$

$$BU = INTEG(BUDOT, BUO)$$

then CSSL can be used to solve a simultaneous equation of forced non-linear vibration and dynamic linear state feedback control which is proposed to control the area of the variable orifice.

The details and procedures of the CSSL computer program for optimal control hydropneumatic recoil mechanism is listed in the following pages.

PROGRAM QUASI-NEWTON USING DAVIDON-FLETCHER-POWELL MODIFICATION RULE
 CONSTANT MR=3.6658, MP=0.06735, RHO=0.78E-4, ALPHA=1.309, PQ=1153.0
 CONSTANT VO=513.0, R=1.68, FR=620.0, FP=700.0
 CONSTANT A1=1.25, A2=0.4536, AR=2.9906, AC=2.4053, AD=11.781
 CONSTANT C1=0.80, C2=0.80, C3=0.80
 CONSTANT C=386.04

TABLE A, 1.56, 0.0, 0.5002, 1.0084, 1.5126, 2.0168, 2.521, 3.0252, 3.5294, ...
 4.0336, 4.5378, 5.042, 5.5462, 6.0504, 6.5546, 7.0588, 7.5630, 8.0672, ...
 8.5714, 9.0756, 9.5798, 10.084, 10.5882, 11.0924, 11.5966, 12.1008, ...
 12.605, 13.1092, 13.6134, 14.1176, 14.6218, 15.126, 15.6302, 16.1344, ...
 16.6386, 17.1428, 17.647, 18.1512, 18.6554, 19.1596, 19.6638, ...
 20.168, 20.6722, 21.1764, 21.6806, 22.1848, 22.689, 23.1932, 23.6974, ...
 24.2016, 24.7058, 25.21, 25.7142, 26.2184, 26.7226, 27.2268, ...
 50.0000, 0.0868, 0.0937, ...
 .0941, .0963, .0995, .1011, .1021, .1025, .1027, .1029, .1028, .1025, .1019, ...
 .1012, .1006, .0996, .0987, .0977, .0967, .0954, .0941, .0929, .0915, .090, ...
 .0885, .0872, .0857, .0841, .0825, .0809, .079, .0771, .0752, .0732, .0713, ...
 .0694, .0675, .0654, .0632, .0610, .0588, .0563, .0538, .0512, .0487, .0458, ...
 .0429, .0398, .0365, .0327, .0287, .0241, .0191, .0133, .0062, 0.0

TABLE B, 1.84, 0.0, 0.0002, 0.0006, 0.001, 0.0014, 0.0018, 0.0022, 0.0026, ...
 0.003, 0.0034, 0.0038, 0.0042, 0.0046, 0.005, 0.0054, 0.0058, 0.0062, 0.0066, ...
 0.0070, ...
 0.0074, 0.0078, 0.0082, 0.0086, 0.009, 0.0094, 0.0098, 0.0102, 0.0106, 0.011, ...
 0.0114, 0.0118, 0.0122, 0.0126, 0.013, 0.0134, 0.0138, 0.0142, 0.0146, 0.015, ...
 0.0154, ...
 0.0158, 0.0162, 0.0166, 0.017, 0.0174, 0.0178, 0.0182, 0.0186, 0.019, 0.0194, ...
 0.0198, 0.0202, 0.0206, 0.021, 0.0214, 0.0218, 0.0222, 0.0226, 0.023, 0.0234, ...
 0.0238, 0.0242, 0.0246, 0.025, 0.0254, 0.0258, 0.0262, 0.0266, 0.027, 0.0274, ...
 0.0278, 0.0282, 0.0286, 0.029, 0.0294, 0.0298, 0.0302, 0.0402, 0.0502, 0.0602, ...
 0.0702, 0.0802, 0.0902, 0.20, ...

2809.6, 4589.6, 7058.6, 18069.0, 35565.0, ...
 68543.0, 126940.0, 219070.0, 337720.0, 446170.0, 496900.0, 476070.00, ...
 411400.0, 335460.0, 267300.0, 211870.0, 168900.0, 136050.0, 110950.0, ...
 91613.0, 76562.0, 62706.0, 55253.0, 47627.0, 41322.0, 36281.0, 32030.0, ...
 29784.0, 28358.0, 27013.0, 25736.0, 24521.0, 23366.0, 22268.0, 21237.0, ...
 20257.0, 19324.0, 18435.0, 17588.0, 16794.0, 16036.0, 15314.0, 14626.0, ...
 13971.0, 13354.0, 12765.0, 12204.0, 11668.0, 11156.0, 10674.0, 10214.0, ...
 9775.1, 9355.5, 8954.5, 8576.1, 8215.1, 7869.6, 7538.8, 7222.5, 6924.1, ...
 6638.1, 6364.5, 6102.9, 5852.1, 5615.3, 5389.0, 5171.7, 4963.0, 4763.3, ...
 4574.0, 4393.7, 4220.4, 4054.0, 3894.2, 3743.0, 3598.0, 3458.6, 3361.8, 578.4, ...
 263.3, 128.1, 63.5, 0.0, 0.0

ALGORITHM 1A=5, 1B=5

CINTERVAL CINT=0.001

NSTEPS NST=6

INITIAL

XDC=0.0

X0=0.0

W1=1.0E-12

W2=0.0

W3=0.0

W4=0.0

W5=1.0E-9

RDPLMX=111.0

BFCF=1.0

```

XFLAG=1.
AN=AC+AD
M1=MR+MP*(1.0+AR/AN)
MQ=MR+MP*(1.0+AR/AN)**2
FQ=FR+FP*AR/AN
MPR=(MP+MR)*G*SIN(ALPHA)
WQ=(MR+MP*(1.0+AR/AN))*G*SIN(ALPHA)
T1=(RHO/2.0)*((AR/A1*C1)**2)
T2=(RHO/2.0)*((AC*AR/(A2*C2*AN))**2)
END
DYNAMIC
TERMT(T.GT.0.2.AND.XDOT.LE.-0.05)
TERMT(T.GT.0.30)
DERIVATIVE MA
T3=(RHO/2.0)*(AD*AR/(C3*AN))**2/((A(X)+SU)**2)
CQ=AR*T1+AR*(AC*T2+AD*T3)/AN
SU=DGK1*X+DGK2*XDOT+DGK3*X*X+DGK4*XDOT*XDOT
DUMBL=WQ/MQ+R(T)*RFCF/MQ-CQ*SWIN(XDOT,-1.0,1.0)*(XDOT**2)/MQ
X2DOT=DUMBL-FQ*SWIN(XDOT,-1.0,1.0)/MQ-AR*PGAS/MQ
XDOT=INTEG(X2DOT,X0)
X=INTEG(XDOT,X0)
PGAS1=V0/(V0-AR*X)
PGAS2=PGAS1**R
PGAS=P0*PGAS2
RDPL=MPR+R(T)*RFCF-W1*X2DOT
PR=(RDPL-FR*SWIN(XDOT,-1.0,1.0))/AR
P1=PR-T1*XDOT*XDOT*SWIN(XDOT,-1.0,1.0)
P3=P1-T3*XDOT*XDOT*SWIN(XDOT,-1.0,1.0)
RDPL2=W1*(RDPL**2)
ORJ1=INTEG(RDPL2,0.0)
XS2=28.2352-X
XS22=W2*(XS2**2)
ORJ2=INTEG(XS22,0.0)
SU2=W3*(0.5-SU)*SU
ORJ3=INTEG(SU2,0.0)
PP3=W4*(P3**2)
ORJ4=INTEG(PP3,0.0)
END
PROCEDURAL(XDOT,X,SU,XFLAG,RDPL,RDPLMX,W2,X2DOT,W3,W4=X2DOT,...
XDOT,X,SU,XFLAG,RDPL,P3,RDPLMX,XS2,SU2,P3)
TTRD=RDPL-RDPLMX
IF(TTRD.LT.0.0) GO TO L4
RDPLMX=RDPL
L4..IF(XDOT.GT.0.) GO TO L1
IF(XFLAG.LE.0.0) GO TO L2
XDOT=0.
X2DOT=0.
X=0.
SU=0.
BU=0.0
GO TO L8
L1..XFLAG=1.0
IF(XS2.GT.0.0) GO TO L5
W2=1.0E+20

```

L5..IF(SD2.GT.0.0) GO TO L6

W3=1.0E+10

L6..IF(P3.GT.0.0) GO TO L2

K4=1.0E+10

L2..IF(XDOT.LT.0.0)GO TO L3

GO TO L8

L8..CONTINUE

END

END

TERMINAL

L3..CONTINUE

VXMAX=X

WWW=OBJ1+OBJ2+OBJ3+OBJ4

PWW2=PWW+OBJ2

PWW3=OBJ1+OBJ2

PWW4=PWW3+OBJ4

PWW=ROPLMX*W9

WDGK1=OGK1

WDGK2=OGK2

WDGK3=OGK3

WDGK4=OGK4

PRINT,WDGK1,WDGK2,WDGK3,WDGK4,PWW,VXMAX,PWW2,PWW3,PWW4,WWW

END

END

DIMENSION GK(4),GRAD(4),H(10),S(16),XM(4),EPS(4)

XCONT=0.0

GK(1)=3.7114E-04

GK(2)=5.1172E-06

GK(3)=4.563E-08

GK(4)=8.73E-12

XM(1)=0.2E-03

XM(2)=0.2E-05

XM(3)=0.2E-08

XM(4)=0.2E-12

DELTA=1.0E-03

EPS(1)=0.5E-06

EPS(2)=0.5E-08

EPS(3)=1.0E-10

EPS(4)=1.0E-14

CALL VA10A(F,4,GK,OBJ,GRAD,H,S,-5.0,XM,DELTA,EPS,1,100,1,IOUT)

N=4

GK(1)=0.0

GK(2)=0.0

GK(3)=0.0

GK(4)=0.0

CALL F(N,GK,OBJ)

XCONT=XCONT+2.0

IF(XCONT.GT.1.0) GO TO L9

END

END

TERMINAL

L9..CONTINUE

END

END

END

Appendix II

Design Data for M-37 Recoil Mechanism

M-37 recoil mechanism is designed for firing zone 7. All the pertinent parameters necessary for simulation of recoil mechanism model developed in Chapter II are given below. The variable area of the groove machined in the floating piston is tabulated in Table 13. Breech forces are tabulated in Tables 14 and 17 for zones 5 and 8. These breech forces are simulated breech force data rather than actual test data. All the design data listed here was provided by Control and Stabilization group of General Rodman Laboratory, Army Weapons Command at Rock Island, Illinois.

m_R - Mass of recoiling parts = 3.6658 lbf sec²/in

m_P - Mass of floating piston = .06735 lbf sec²/in

P_O - Initial gas pressure = 1153 psi

V_O - Initial volume of gas = 513 in³

R - Gas constant = 1.68

A_R - Recoil piston area = 2.9906 in²

A_C - Control rod area = 2.4053 in²

A_D - Floating piston area = 11.781 in²

$C_1=C_2=C_3$ - Discharge coefficients = .8

A_1 - Area of orifice 1 = 1.25 in²

A_2 - Area of orifice 2 = .4536 in²

TABLE 13- AREA OF VARIABLE ORIFICE A3

X	A3(X)	X	A3(X)
0.00000	.08680	15.12600	.07900
.50420	.09370	15.63020	.07710
1.00840	.09410	16.13440	.07520
1.51260	.09630	16.63860	.07320
2.01680	.09950	17.14280	.07130
2.52100	.10110	17.64700	.06940
3.02520	.10210	18.15120	.06750
3.52940	.10250	18.65540	.06540
4.03360	.10270	19.15960	.06320
4.53780	.10290	19.66380	.06100
5.04200	.10280	20.16800	.05880
5.54620	.10250	20.67220	.05630
6.05040	.10190	21.17640	.05380
6.55460	.10120	21.68060	.05120
7.05880	.10060	22.18480	.04870
7.56300	.09960	22.68900	.04580
8.06720	.09870	23.19320	.04290
8.57140	.09770	23.69740	.03980
9.07560	.09670	24.20160	.03650
9.57980	.09540	24.70580	.03270
10.08400	.09410	25.21000	.02870
10.58820	.09290	25.71420	.02410
11.09240	.09150	26.21840	.01910
11.59660	.09000	26.72260	.01330
12.10080	.08850	27.22680	.00620
12.60500	.08720	27.73100	0.00000
13.10920	.08570	28.23520	0.00000
13.61340	.08410	28.73940	0.00000
14.11760	.08250	29.24360	0.00000
14.62180	.08090		

TABLE 14- BREECH FORCE FOR ZONE 5

T	B(T)	T	B(T)
0.00000	159.0	.03600	2426.0
.00050	246.0	.03800	2069.0
.00100	313.0	.04000	1769.0
.00150	938.0	.04200	1517.0
.00200	1724.0	.04400	1304.0
.00250	3118.0	.04600	1123.0
.00300	5642.0	.04800	969.0
.00350	10127.0	.05000	829.0
.00400	18001.0	.05200	727.0
.00450	31434.0	.05400	622.0
.00500	53257.0	.05600	530.0
.00550	85873.0	.05800	480.0
.00600	128386.0	.06000	419.0
.00650	173036.0	.06200	367.0
.00700	206142.0	.06400	322.0
.00748	217035.0	.06600	283.0
.00750	217008.0	.06800	249.0
.00800	205962.0	.07000	220.0
.00850	181687.0	.07200	194.0
.00900	153385.0	.07400	172.0
.00950	126693.0	.07600	152.0
.01000	103872.0	.07800	125.0
.01050	85256.0	.08000	120.0
.01100	70386.0	.08200	107.0
.01200	49196.0	.08400	95.0
.01250	41695.0	.08600	85.0
.01300	35654.0	.08800	76.0
.01350	30747.0	.09000	68.0
.01400	26725.0	.09200	61.0
.01450	23400.0	.09400	54.0
.01500	20625.0	.09600	49.0
.01550	18298.0	.09800	44.0
.01600	16324.0	.10000	39.0
.01650	14641.0	.10200	36.0
.01700	13195.0	.10400	32.0
.01734	12315.0	.10600	29.0
.01750	12135.0	.10800	26.0
.01800	11575.0	.11000	24.0
.02000	9605.0	.11200	21.0
.02200	7996.0	.11400	19.0
.02400	6683.0	.11600	18.0
.02600	5602.0	.11800	16.0
.02800	4711.0	.20000	0.0
.03000	3973.0		
.03200	3361.0		
.03400	2852.0		

TABLE 15-- BREACH FORCE FOR ZONE 6

T	R(T)	T	R(T)
0.00000	384.0	.03500	2545.0
.00050	706.0	.03700	2135.0
.00100	1491.0	.03900	1797.0
.00150	2937.0	.04100	1517.0
.00200	5800.0	.04200	1285.0
.00250	11543.0	.04500	1091.0
.00300	21947.0	.04700	929.0
.00350	41614.0	.04900	793.0
.00400	76017.0	.05100	670.0
.00450	130033.0	.05300	582.0
.00500	200301.0	.05500	500.0
.00550	267029.0	.05700	431.0
.00600	302300.0	.05900	373.0
.00616	304395.0	.06100	323.0
.00650	295349.0	.06300	280.0
.00700	259560.0	.06500	243.0
.00750	214473.0	.06700	212.0
.00800	172277.0	.06900	185.0
.00850	137329.0	.07100	162.0
.00900	109863.0	.07300	142.0
.00950	88691.0	.07500	124.0
.01000	72415.0	.07700	109.0
.01050	59835.0	.07900	96.0
.01100	50018.0	.08100	85.0
.01150	42274.0	.08300	75.0
.01400	20885.0	.08500	66.0
.01431	19380.0	.08700	59.0
.01450	18986.0	.08900	52.0
.01500	17983.0	.09100	46.0
.01700	14514.0	.09300	41.0
.01900	11769.0	.09500	37.0
.02100	9586.0	.09700	33.0
.02300	7841.0	.09900	29.0
.02500	6440.0	.10100	26.0
.02700	5310.0	.10300	24.0
.02900	4395.0	.20000	0.0
.03100	3651.0		
.03300	3043.0		

TABLE 16- BREECH FORCE FOR ZONE 7

T	B(T)	T	B(T)
0.00000	2809.6	.01700	13971.0
.00020	4589.6	.01740	13354.0
.00060	9053.6	.01780	12765.0
.00100	18069.0	.01820	12204.0
.00140	35565.0	.01860	11668.0
.00180	68543.0	.01900	11156.0
.00220	126940.0	.01940	10674.0
.00260	219070.0	.01980	10214.0
.00300	337720.0	.02020	9775.1
.00340	446170.0	.02060	9355.5
.00380	496900.0	.02100	8954.5
.00420	476070.0	.02140	8576.1
.00460	411400.0	.02180	8215.1
.00500	335660.0	.02220	7869.6
.00540	267300.0	.02260	7538.8
.00580	211870.0	.02300	7222.5
.00620	168900.0	.02340	6924.1
.00660	136050.0	.02380	6626.1
.00700	110950.0	.02420	6364.5
.00740	91613.0	.02460	6102.9
.00780	76562.0	.02500	5852.1
.00820	64706.0	.02540	5615.3
.00860	55253.0	.02580	5389.0
.00900	47627.0	.02620	5171.7
.00940	41322.0	.02660	4963.0
.00980	36281.0	.02700	4763.3
.01020	32030.0	.02740	4574.0
.01060	29784.0	.02780	4393.7
.01100	28356.0	.02820	4220.4
.01140	27013.0	.02860	4054.0
.01180	25736.0	.02900	3894.2
.01220	24521.0	.02940	3743.0
.01260	23366.0	.02980	3598.0
.01300	22263.0	.03020	3458.6
.01340	21237.0	.04020	1361.8
.01380	20257.0	.05020	578.4
.01420	19324.0	.06020	263.3
.01460	18435.0	.07020	126.1
.01500	17588.0	.08020	63.5
.01540	16794.0	.09020	0.0
.01580	16036.0	.20000	0.0
.01620	15314.0		
.01660	14626.0		

TABLE 17 - BREACH FORCE FOR ZONE 8

T	B(T)	T	B(T)
0.00000	3090.6	.01700	15268.1
.00020	5048.6	.01740	14689.4
.00060	9964.5	.01780	14041.5
.00100	19875.9	.01820	13424.4
.00140	39121.5	.01860	12824.8
.00180	75397.3	.01900	12271.6
.00220	139634.0	.01940	11741.4
.00260	240977.0	.01980	11235.4
.00300	371491.9	.02020	10752.6
.00340	490786.9	.02060	10291.0
.00380	546589.9	.02100	9849.9
.00420	523676.9	.02140	9423.7
.00460	452539.9	.02180	9036.6
.00500	369225.9	.02220	8656.6
.00540	294029.9	.02260	8292.7
.00580	233057.0	.02300	7944.7
.00620	135790.0	.02340	7616.5
.00660	147635.0	.02380	7301.9
.00700	122045.0	.02420	7000.9
.00740	100774.3	.02460	6713.2
.00780	84218.2	.02500	6427.3
.00820	71176.6	.02540	6176.8
.00860	60778.3	.02580	5927.9
.00900	52389.7	.02620	5688.9
.00940	45454.2	.02660	5459.3
.00980	39909.1	.02700	5239.6
.01020	35233.0	.02740	5021.4
.01060	32762.4	.02780	4833.1
.01100	31193.8	.02820	4642.4
.01140	29714.3	.02860	4459.4
.01180	28300.6	.02900	4283.6
.01220	26973.1	.02940	4117.3
.01260	25702.6	.02980	3957.8
.01300	24494.8	.03020	3804.5
.01340	23360.7	.04020	1498.0
.01380	22282.7	.05020	626.2
.01420	21256.4	.06020	289.6
.01460	20278.5	.07020	140.9
.01500	19346.8	.08020	69.8
.01540	18473.4	.09020	0.0
.01580	17639.6	.20000	0.0
.01620	16845.4		
.01660	16088.6		

Appendix III

Recoil Acceleration Data for the M-37 Recoil Mechanism

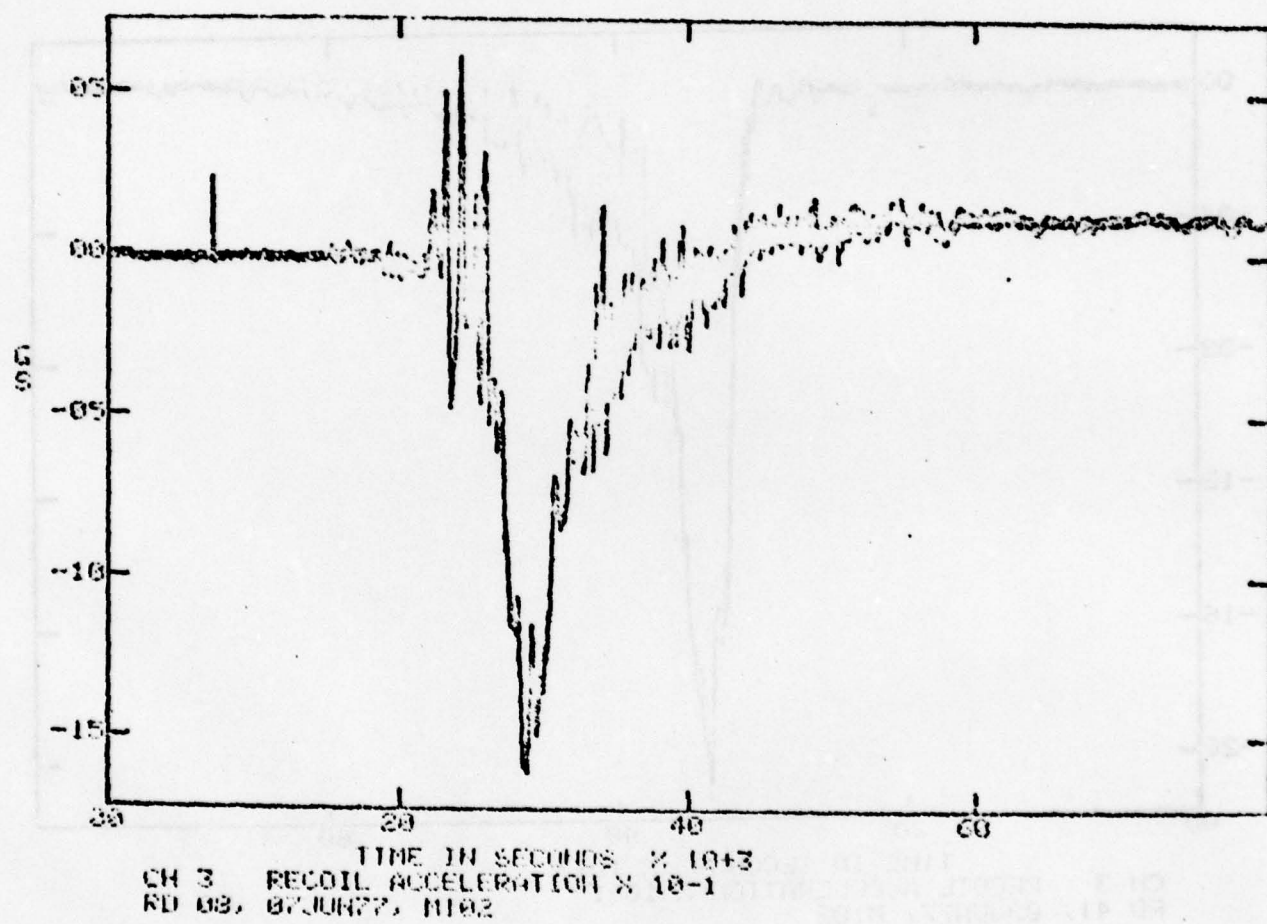


Figure 17 Recoil Acceleration of Zone 5

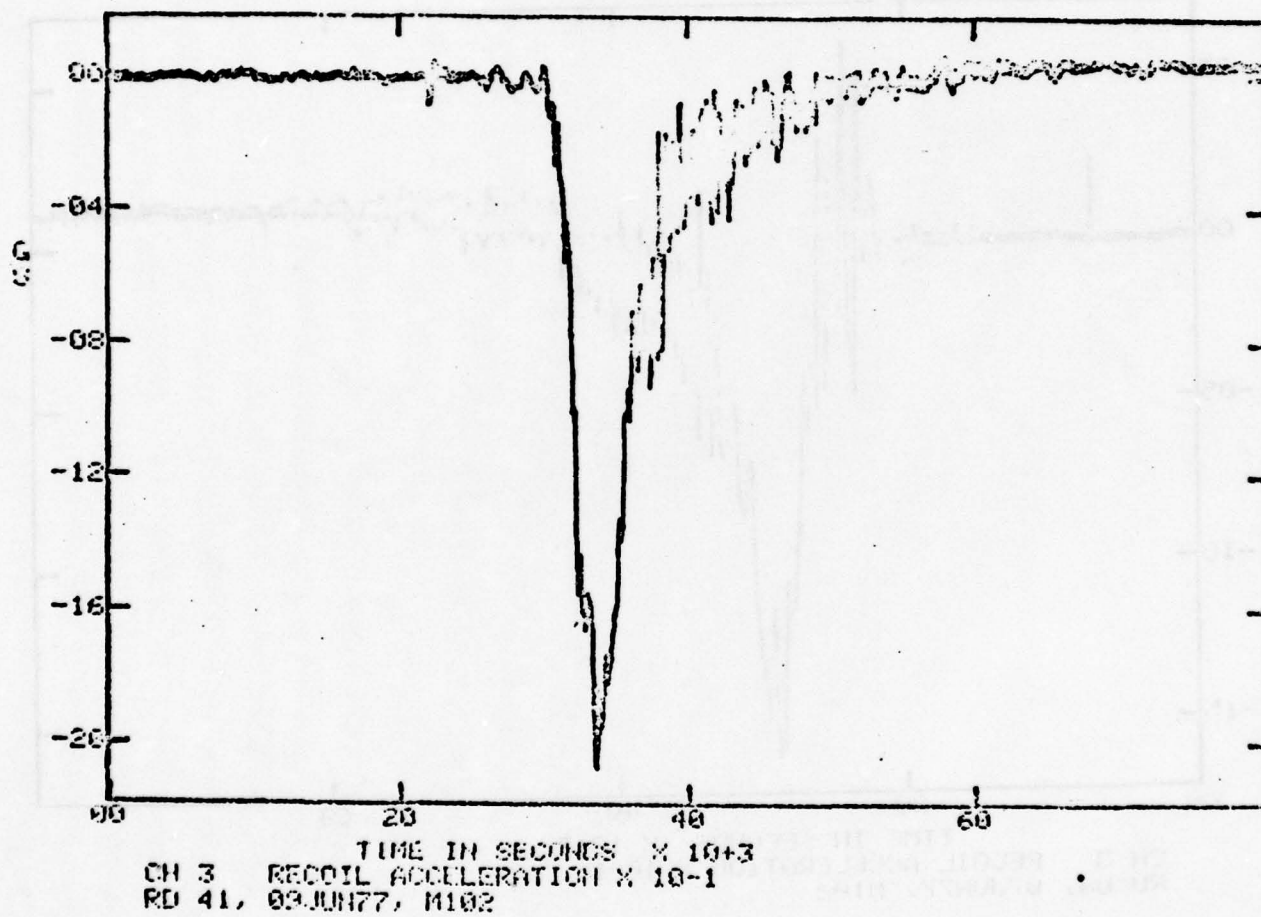


Figure 18 Recoil Acceleration of Zone 6

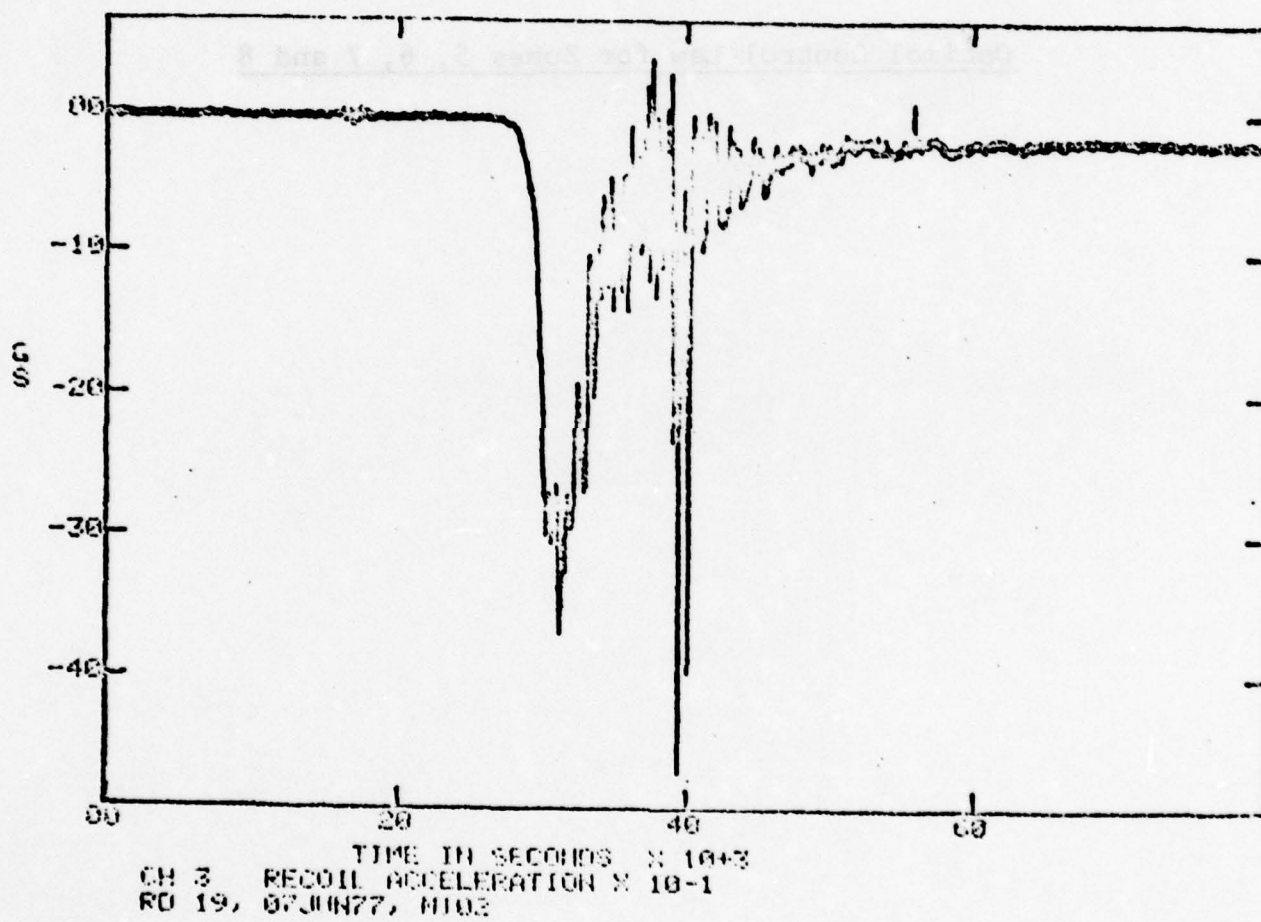


Figure 19 Recoil Acceleration of Zone 7

Appendix IV

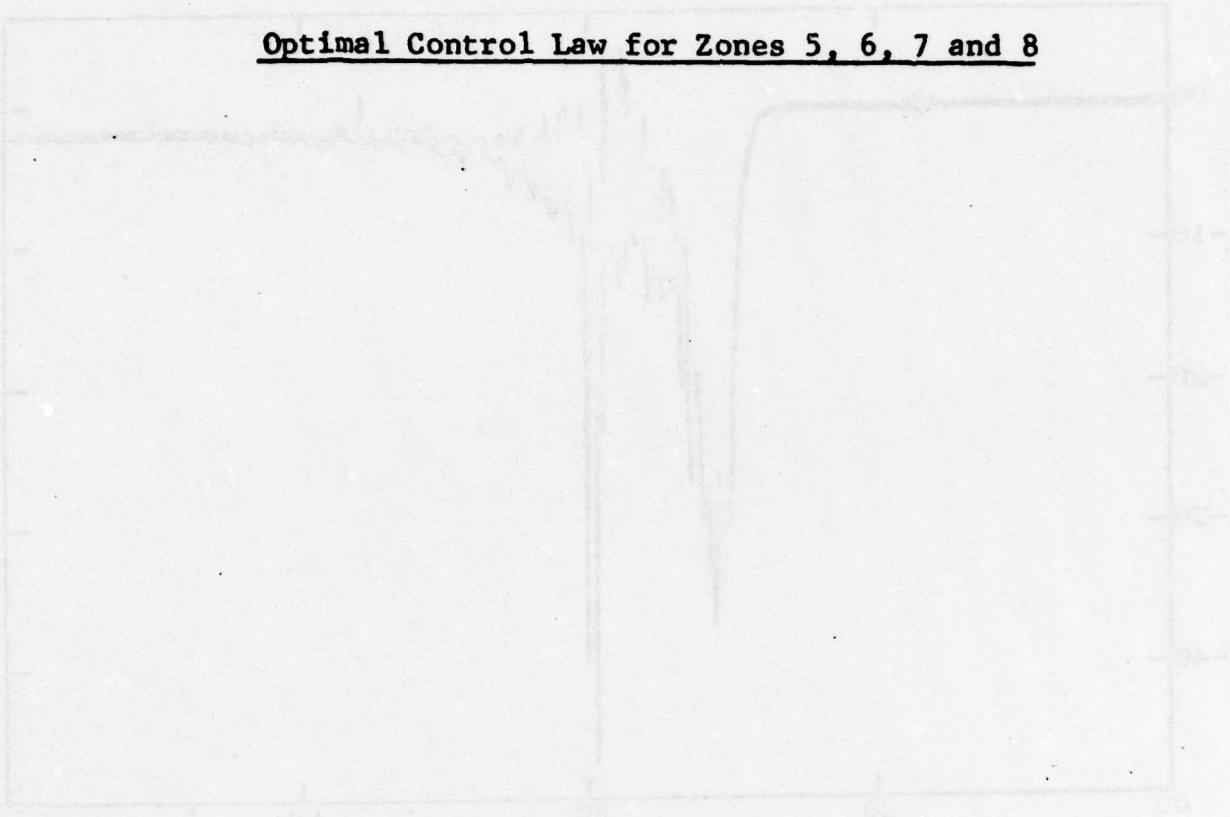
Optimal Control Law for Zones 5, 6, 7 and 8

Figure 19. Acceleration of Zone 7

Table 18. Optimal Control Law of Zone 5.

T (sec)	γ_u (in ²)	s_u (in ²)	RDPL (lbf)
1.000000-03	.100629	0.000000	2671.77
2.000000-03	9.923132-02	0.000000	2677.29
3.000000-03	9.925003-02	1.245882-02	4223.31
4.000000-03	.100419	1.361512-02	4280.27
4.999999-03	.100711	1.383035-02	4451.05
5.999998-03	.101632	1.447563-02	4884.23
6.999998-03	.104003	1.611242-02	5661.04
7.999997-03	.108378	1.904762-02	6547.17
8.999996-03	.113880	2.239585-02	7233.90
9.999995-03	.118718	2.499065-02	7673.79
1.099999-02	.120558	2.665280-02	8048.96
1.199999-02	.122166	2.807037-02	8296.11
1.299999-02	.124184	2.900424-02	8407.01
1.399999-02	.125745	2.942326-02	8473.79
1.499999-02	.127682	2.967558-02	8466.50
1.599999-02	.129255	2.964805-02	8448.60
1.699998-02	.130050	2.958043-02	8455.49
1.799998-02	.130855	2.960645-02	8448.80
1.899998-02	.131374	2.958117-02	8453.79
1.999998-02	.131796	2.960003-02	8457.06
2.099997-02	.132027	2.961237-02	8465.81
2.199997-02	.132213	2.964546-02	8471.05
2.299997-02	.132342	2.966523-02	8474.62
2.399997-02	.132466	2.967872-02	8473.42
2.499996-02	.132571	2.967420-02	8469.22
2.599996-02	.132505	2.965834-02	8471.28
2.699996-02	.132442	2.966610-02	8469.75
2.799996-02	.132272	2.966034-02	8471.25
2.899995-02	.132063	2.966598-02	8472.32
2.999995-02	.131740	2.967005-02	8477.33
3.099995-02	.131408	2.968896-02	8480.54
3.199994-02	.131041	2.970109-02	8483.35
3.299994-02	.130709	2.971173-02	8482.09
3.399993-02	.130382	2.970694-02	8478.43
3.499993-02	.129884	2.969311-02	8483.55
3.599992-02	.129371	2.971248-02	8487.80
3.699992-02	.128904	2.972851-02	8487.72
3.799991-02	.128480	2.972824-02	8486.66
3.899990-02	.127902	2.972423-02	8487.65
3.999990-02	.127384	2.972795-02	8486.72
4.099989-02	.126862	2.972445-02	8484.99
4.199989-02	.126315	2.971790-02	8483.64
4.299988-02	.125643	2.971280-02	8489.15
4.399988-02	.125001	2.973363-02	8491.84
4.499987-02	.124353	2.974380-02	8494.11

T	YU	SU	RDPL
4.599987-02	.123718	2.975238-02	8494.67
4.699986-02	.123119	2.975450-02	8492.21
4.799986-02	.122491	2.974520-02	8490.88
4.899985-02	.121794	2.974018-02	8493.24
4.999985-02	.121106	2.974908-02	8494.37
5.099984-02	.120379	2.975337-02	8497.41
5.199983-02	.119664	2.976485-02	8499.08
5.299983-02	.118949	2.977115-02	8500.23
5.399982-02	.118242	2.977549-02	8500.31
5.499982-02	.117626	2.977581-02	8494.06
5.599981-02	.116991	2.975218-02	8488.60
5.699981-02	.116276	2.973154-02	8487.90
5.799980-02	.115578	2.972890-02	8485.65
5.899980-02	.114843	2.972039-02	8485.39
5.999979-02	.114111	2.971942-02	8484.51
6.099979-02	.113383	2.971611-02	8483.05
6.199978-02	.112658	2.971058-02	8480.99
6.299977-02	.111937	2.970278-02	8478.35
6.399977-02	.111219	2.969283-02	8475.13
6.499976-02	.110491	2.968067-02	8472.31
6.599976-02	.109653	2.967000-02	8476.46
6.699975-02	.108849	2.968568-02	8478.08
6.799975-02	.108042	2.969179-02	8479.56
6.899974-02	.107241	2.969741-02	8480.33
6.999974-02	.106444	2.970029-02	8480.51
7.099973-02	.105652	2.970098-02	8480.08
7.199973-02	.104863	2.969936-02	8479.17
7.299972-02	.104040	2.969592-02	8480.35
7.399972-02	.103282	2.970039-02	8480.20
7.499971-02	.102450	2.969982-02	8477.96
7.599970-02	.101684	2.969133-02	8474.34
7.699970-02	.100919	2.967766-02	8470.33
7.799969-02	.100160	2.966253-02	8465.68
7.899969-02	9.940572-02	2.964494-02	8460.45
7.999968-02	9.865599-02	2.962517-02	8454.62
8.099968-02	9.791098-02	2.960314-02	8448.20
8.199967-02	9.717075-02	2.957890-02	8441.20
8.299967-02	9.637030-02	2.955243-02	8438.31
8.399966-02	9.558346-02	2.954152-02	8434.18
8.499966-02	9.479364-02	2.952593-02	8430.03
8.599965-02	9.398125-02	2.951023-02	8427.28
8.699964-02	9.318229-02	2.949987-02	8423.31
8.799964-02	9.238676-02	2.948485-02	8418.83
8.899963-02	9.159742-02	2.946792-02	8413.64
8.999963-02	9.081350-02	2.944833-02	8407.81

T	YU	SU	RDPL
9.099962-02	9.003521-02	2.942627-02	8401.30
9.199962-02	8.926249-02	2.940169-02	8394.13
9.299961-02	8.849534-02	2.937460-02	8386.29
9.399961-02	8.767765-02	2.934498-02	8382.07
9.499960-02	8.683908-02	2.932903-02	8379.20
9.599960-02	8.601481-02	2.931820-02	8374.99
9.699959-02	8.519463-02	2.930228-02	8370.21
9.799958-02	8.438148-02	2.928423-02	8364.63
9.899958-02	8.357447-02	2.926315-02	8358.32
9.999957-02	8.276322-02	2.923928-02	8352.08
.101000	8.194135-02	2.921572-02	8346.44
.102000	8.113125-02	2.919440-02	8339.61
.103000	8.032779-02	2.916859-02	8331.98
.104000	7.955880-02	2.913977-02	8321.31
.105000	7.878743-02	2.909944-02	8310.57
.106000	7.802493-02	2.905886-02	8298.85
.107000	7.719636-02	2.901458-02	8292.30
.108000	7.635528-02	2.898981-02	8286.50
.109000	7.552761-02	2.896792-02	8279.33
.110000	7.470528-02	2.894083-02	8271.45
.111000	7.389077-02	2.891104-02	8262.63
.112000	7.308324-02	2.887771-02	8252.93
.113000	7.228292-02	2.884109-02	8242.35
.113999	7.147513-02	2.880109-02	8232.13
.114999	7.064253-02	2.876247-02	8223.76
.115999	6.982811-02	2.873084-02	8213.53
.116999	6.901785-02	2.869220-02	8202.65
.117999	6.820110-02	2.865109-02	8192.05
.118999	6.736335-02	2.861102-02	8183.00
.119999	6.654332-02	2.857685-02	8172.09
.120999	6.572805-02	2.853562-02	8160.45
.121999	6.491126-02	2.849164-02	8148.65
.122999	6.400252-02	2.844707-02	8144.90
.123999	6.313773-02	2.843289-02	8136.85
.124999	6.227022-02	2.840245-02	8128.73
.125999	6.141598-02	2.837176-02	8119.05
.126999	6.054485-02	2.833520-02	8110.65
.127999	5.967462-02	2.830346-02	8101.83
.128999	5.881701-02	2.827016-02	8091.48
.129999	5.796773-02	2.823102-02	8079.97
.130999	5.712823-02	2.818754-02	8067.15
.131999	5.621672-02	2.813912-02	8061.12
.132999	5.531092-02	2.811630-02	8054.18
.133999	5.441791-02	2.809009-02	8045.60
.134999	5.353490-02	2.805769-02	8035.65

T	YU	SU	RDPL
.135999	5.266280-02	2.802007-02	8024.18
.136999	5.177717-02	2.797676-02	8013.76
.137999	5.086578-02	2.793738-02	8005.68
.138999	4.998069-02	2.790683-02	7994.40
.139999	4.910098-02	2.786423-02	7982.13
.140999	4.823491-02	2.781786-02	7967.94
.141999	4.737897-02	2.776423-02	7952.18
.142999	4.649121-02	2.770468-02	7939.56
.143999	4.555303-02	2.765699-02	7932.22
.144999	4.465486-02	2.762927-02	7919.87
.145999	4.375777-02	2.758262-02	7906.92
.146999	4.287836-02	2.753367-02	7891.38
.147999	4.200913-02	2.747497-02	7874.11
.148999	4.115320-02	2.740968-02	7854.67
.149999	4.023557-02	2.733625-02	7842.33
.150999	3.926956-02	2.728962-02	7835.55
.151999	3.834872-02	2.726399-02	7822.56
.152999	3.742857-02	2.721490-02	7808.90
.153999	3.653000-02	2.716329-02	7791.80
.154999	3.564246-02	2.709870-02	7772.58
.155999	3.477081-02	2.702605-02	7750.50
.156999	3.391227-02	2.694263-02	7725.88
.157999	3.306792-02	2.684960-02	7698.50
.158999	3.293245-02	2.674616-02	7572.97
.159999	3.244450-02	2.627183-02	7497.39
.160999	3.214576-02	2.598626-02	7398.94
.161999	3.176101-02	2.561425-02	7313.49
.162999	3.142583-02	2.529138-02	7223.98
.163999	3.107574-02	2.495317-02	7138.31
.164999	3.074059-02	2.462948-02	7053.02
.165999	3.040727-02	2.430721-02	6969.64
.166999	3.008157-02	2.399216-02	6887.59
.167999	2.976129-02	2.368212-02	6807.15
.168999	2.944749-02	2.337817-02	6728.26
.169999	2.913996-02	2.308010-02	6651.01
.170999	2.883899-02	2.278821-02	6575.43
.171999	2.854489-02	2.250262-02	6501.56
.172999	2.825727-02	2.222353-02	6429.47
.173999	2.797690-02	2.195112-02	6359.20
.174999	2.770378-02	2.168560-02	6290.81
.175999	2.743812-02	2.142719-02	6224.36
.176999	2.718013-02	2.117610-02	6159.91
.177999	2.693007-02	2.093259-02	6097.53
.178999	2.668816-02	2.069689-02	6037.29
.179999	2.645467-02	2.046926-02	5979.26

T	YU	SU	RDPL
.180999	2.622986-02	2.025000-02	5923.52
.181999	2.601403-02	2.003937-02	5870.14
.182999	2.580746-02	1.983768-02	5819.21
.183999	2.561046-02	1.964524-02	5770.81
.184999	2.542334-02	1.946237-02	5725.03
.185999	2.524643-02	1.928939-02	5681.96
.186999	2.508007-02	1.912664-02	5641.68
.187999	2.492457-02	1.897445-02	5604.29
.188999	2.478029-02	1.883318-02	5569.88
.189999	2.464756-02	1.870314-02	5538.53
.190999	2.452672-02	1.858470-02	5510.33
.191999	2.441808-02	1.847816-02	5485.37
.192999	2.432197-02	1.838384-02	5463.73
.193999	2.423867-02	1.830205-02	5445.47
.194999	2.416847-02	1.823306-02	5430.66
.195999	2.411161-02	1.817711-02	5419.37
.196999	2.406831-02	1.813444-02	5411.63
.197999	2.403875-02	1.810521-02	5407.50

Table 19. Optimal Control Law of Zone 6.

T (sec)	y_U (in ²)	s_U (in ²)	RDPL (lbf)
1.000000-03	9.189969-02	0.000000	2676.24
2.000000-03	9.139199-02	4.593426-03	4224.05
3.000000-03	9.183666-02	5.022364-03	4298.69
4.000000-03	9.203992-02	5.126380-03	4566.85
4.999999-03	9.280726-02	5.500061-03	5372.94
5.999998-03	9.503433-02	6.623384-03	6992.79
6.999998-03	9.942145-02	8.880692-03	8884.60
7.999997-03	.105094	1.151699-02	10184.5
8.999996-03	.107233	1.332846-02	11198.0
9.999995-03	.109045	1.474081-02	11805.9
1.099999-02	.111248	1.558793-02	12057.5
1.199999-02	.113348	1.593860-02	12141.7
1.299999-02	.115552	1.605585-02	12106.8
1.399999-02	.116565	1.600728-02	12123.9
1.499999-02	.117460	1.603100-02	12108.0
1.599999-02	.118109	1.600889-02	12105.3
1.699998-02	.118375	1.600510-02	12131.3
1.799998-02	.118612	1.604139-02	12143.8
1.899998-02	.118765	1.605885-02	12150.5
1.999998-02	.118910	1.606815-02	12144.7
2.099997-02	.118921	1.606004-02	12143.1
2.199997-02	.118839	1.605779-02	12142.9
2.299997-02	.118635	1.605750-02	12147.7
2.399997-02	.118315	1.606422-02	12158.9
2.499996-02	.117918	1.607990-02	12171.5
2.599996-02	.117465	1.609741-02	12184.1
2.699996-02	.117058	1.611489-02	12183.8
2.799996-02	.116621	1.611450-02	12181.6
2.899995-02	.115955	1.611147-02	12203.6
2.999995-02	.115368	1.614219-02	12210.4
3.099995-02	.114781	1.615167-02	12212.6
3.199994-02	.114132	1.615469-02	12218.9
3.299994-02	.113494	1.616344-02	12219.9
3.399993-02	.112848	1.616490-02	12218.6
3.499993-02	.112015	1.616303-02	12239.2
3.599992-02	.111218	1.619180-02	12252.1
3.699992-02	.110417	1.620973-02	12262.9
3.799991-02	.109672	1.622471-02	12263.2
3.899990-02	.108895	1.622523-02	12265.9
3.999990-02	.108036	1.622896-02	12277.9
4.099989-02	.107157	1.624566-02	12290.6
4.199989-02	.106266	1.626340-02	12303.5
4.299988-02	.105381	1.628131-02	12313.5
4.399988-02	.104540	1.629525-02	12315.7
4.499987-02	.103775	1.629833-02	12305.3

T	YU	SU	RDPL
4.599987-02	.102924	1.628391-02	12306.2
4.699986-02	.102054	1.628504-02	12308.4
4.799986-02	.101142	1.628823-02	12315.9
4.899985-02	.100240	1.629864-02	12320.5
4.999985-02	9.934284-02	1.630509-02	12323.4
5.099984-02	9.845196-02	1.630907-02	12324.3
5.199983-02	9.756698-02	1.631027-02	12323.2
5.299983-02	9.659049-02	1.630886-02	12335.5
5.399982-02	9.557600-02	1.632594-02	12352.8
5.499982-02	9.457909-02	1.635011-02	12366.6
5.599981-02	9.358775-02	1.636931-02	12378.8
5.699981-02	9.260469-02	1.638623-02	12388.8
5.799980-02	9.162936-02	1.640025-02	12396.9
5.899980-02	9.061199-02	1.641153-02	12411.3
5.999979-02	8.961398-02	1.643153-02	12421.8
6.099979-02	8.867137-02	1.644617-02	12422.3
6.199978-02	8.772560-02	1.644688-02	12422.7
6.299977-02	8.679046-02	1.644743-02	12420.6
6.399977-02	8.586269-02	1.644454-02	12416.6
6.499976-02	8.494311-02	1.643902-02	12410.6
6.599976-02	8.403152-02	1.643065-02	12402.6
6.699975-02	8.304468-02	1.641950-02	12407.5
6.799975-02	8.207711-02	1.642623-02	12408.3
6.899974-02	8.108214-02	1.642741-02	12413.6
6.999974-02	8.009405-02	1.643481-02	12417.2
7.099973-02	7.911613-02	1.643974-02	12418.3
7.199973-02	7.814751-02	1.644132-02	12417.2
7.299972-02	7.718841-02	1.643976-02	12413.7
7.399972-02	7.623874-02	1.643497-02	12408.0
7.499971-02	7.529850-02	1.642693-02	12399.9
7.599970-02	7.425203-02	1.641562-02	12412.1
7.699970-02	7.323864-02	1.643271-02	12417.4
7.799969-02	7.223000-02	1.644012-02	12421.3
7.899969-02	7.123385-02	1.644556-02	12422.2
7.999968-02	7.024798-02	1.644681-02	12420.5
8.099968-02	6.923411-02	1.644445-02	12424.3
8.199967-02	6.824279-02	1.644963-02	12422.8
8.299967-02	6.727421-02	1.644754-02	12415.9
8.399966-02	6.633540-02	1.643792-02	12402.0
8.499966-02	6.540135-02	1.641859-02	12386.5
8.599965-02	6.438078-02	1.639706-02	12390.0
8.699964-02	6.335804-02	1.640182-02	12393.4
8.799964-02	6.235218-02	1.640668-02	12392.6
8.899963-02	6.135713-02	1.640554-02	12388.8
8.999963-02	6.037470-02	1.640023-02	12381.5

T	YU	SU	RDPL
9.099962-02	5.940424-02	1.639008-02	12370.9
9.199962-02	5.838866-02	1.637523-02	12370.7
9.299961-02	5.739776-02	1.637500-02	12364.0
9.399961-02	5.641569-02	1.636566-02	12354.6
9.499960-02	5.540228-02	1.635252-02	12352.7
9.599960-02	5.440231-02	1.634987-02	12346.8
9.699959-02	5.341590-02	1.634171-02	12336.9
9.799958-02	5.238452-02	1.632786-02	12338.6
9.899958-02	5.130483-02	1.633023-02	12353.2
9.999957-02	5.026407-02	1.635057-02	12356.4
.101000	4.922997-02	1.635507-02	12357.4
.102000	4.817050-02	1.635649-02	12365.4
.103000	4.713475-02	1.636766-02	12366.1
.104000	4.611189-02	1.636855-02	12362.3
.105000	4.510611-02	1.636334-02	12352.8
.106000	4.397525-02	1.635006-02	12382.5
.107000	4.291926-02	1.639140-02	12388.1
.108000	4.185641-02	1.639917-02	12395.4
.109000	4.082279-02	1.640944-02	12392.5
.110000	3.974034-02	1.640534-02	12406.3
.111000	3.868305-02	1.642457-02	12410.8
.112000	3.764197-02	1.643090-02	12408.9
.113000	3.662100-02	1.642825-02	12398.8
.113999	3.561769-02	1.641419-02	12381.2
.114999	3.449580-02	1.638959-02	12410.5
.115999	3.345653-02	1.643041-02	12406.0
.116999	3.240400-02	1.642423-02	12406.4
.117999	3.139186-02	1.642470-02	12388.5
.118999	3.038804-02	1.639979-02	12365.8
.119999	2.935358-02	1.636821-02	12357.0
.120999	2.822527-02	1.635589-02	12392.8
.121999	2.720044-02	1.640583-02	12377.6
.122999	2.614567-02	1.638461-02	12376.4
.123999	2.515180-02	1.638289-02	12341.9
.124999	2.415234-02	1.633485-02	12308.5
.125999	2.319552-02	1.628827-02	12249.2
.126999	2.240265-02	1.620569-02	12088.8
.127999	2.216300-02	1.598217-02	11613.7
.128999	2.148563-02	1.532007-02	11397.5
.129999	2.116986-02	1.501885-02	10998.3
.130999	2.059962-02	1.446245-02	10746.4
.131999	2.023549-02	1.411147-02	10402.8
.132999	1.974418-02	1.363266-02	10134.9
.133999	1.935898-02	1.325930-02	9829.71
.134999	1.892248-02	1.283404-02	9560.82

T	YU	SU	RDPL
.135999	1.853714-02	1.245933-02	9282.38
.136999	1.813906-02	1.207131-02	9021.97
.137999	1.776666-02	1.170842-02	8763.90
.138999	1.739808-02	1.134879-02	8516.89
.139999	1.704543-02	1.100458-02	8276.56
.140999	1.670258-02	1.066966-02	8045.20
.141999	1.637273-02	1.034727-02	7821.92
.142999	1.605460-02	1.003612-02	7607.29
.143999	1.574897-02	9.737014-03	7401.24
.144999	1.545575-02	9.449881-03	7204.00
.145999	1.517522-02	9.175022-03	7015.70
.146999	1.490757-02	8.912620-03	6836.52
.147999	1.465301-02	8.662926-03	6666.64
.148999	1.441181-02	8.426194-03	6506.27
.149999	1.418422-02	8.202707-03	6355.61
.150999	1.397053-02	7.992766-03	6214.91
.151999	1.377107-02	7.796696-03	6084.42
.152999	1.358617-02	7.614848-03	5964.39
.153999	1.341619-02	7.447590-03	5855.12
.154999	1.326152-02	7.295314-03	5756.89
.155999	1.312255-02	7.158426-03	5670.00
.156999	1.299970-02	7.037344-03	5594.76
.157999	1.289339-02	6.932493-03	5531.47
.158999	1.280403-02	6.844296-03	5480.42
.159999	1.273202-02	6.773161-03	5441.90
.160999	1.267776-02	6.719474-03	5416.14
.161999	1.264159-02	6.683582-03	5403.37

Table 20. Optimal Control Law of Zone 7.

T (sec)	v_u (in ²)	s_u (in ²)	RDPL (lbf)
1.000000-03	9.753114-02	1.072441-02	4280.35
2.000000-03	9.760680-02	1.071817-02	4662.47
3.000000-03	9.793906-02	1.057357-02	6175.88
4.000000-03	9.861749-02	9.586117-03	9658.63
4.999999-03	.100156	7.809828-03	13773.0
5.999998-03	.100266	6.383099-03	17025.0
6.999998-03	.100058	5.505600-03	19146.3
7.999997-03	.101335	5.017384-03	19783.3
8.999996-03	.103958	5.039061-03	19058.1
9.999995-03	.105350	4.824982-03	19151.2
1.099999-02	.106279	4.705527-03	19162.6
1.199999-02	.106853	4.627137-03	19194.9
1.299999-02	.107105	4.573357-03	19256.8
1.399999-02	.107243	4.541490-03	19298.9
1.499999-02	.107400	4.528665-03	19291.3
1.599999-02	.107362	4.532521-03	19298.6
1.699998-02	.107186	4.551089-03	19310.1
1.799998-02	.106844	4.582651-03	19336.9
1.899998-02	.106356	4.625742-03	19376.9
1.999998-02	.105829	4.678945-03	19403.7
2.099997-02	.105389	4.740689-03	19385.1
2.199997-02	.104658	4.809914-03	19429.5
2.299997-02	.103965	4.886690-03	19443.5
2.399997-02	.103267	4.968109-03	19446.2
2.499996-02	.102538	5.054273-03	19444.4
2.599996-02	.101811	5.144406-03	19429.5
2.699996-02	.100862	5.238041-03	19468.6
2.799996-02	9.992411-02	5.334788-03	19494.0
2.899995-02	9.903962-02	5.434020-03	19494.5
2.999995-02	9.816495-02	5.535120-03	19484.0
3.099995-02	9.718423-02	5.637097-03	19501.3
3.199994-02	9.614038-02	5.738390-03	19537.5
3.299994-02	9.510406-02	5.839002-03	19570.7
3.399993-02	9.412664-02	5.938878-03	19584.8
3.499993-02	9.326031-02	6.037757-03	19563.4
3.599992-02	9.225964-02	6.135691-03	19584.1
3.699992-02	9.122193-02	6.232842-03	19616.1
3.799991-02	9.016772-02	6.329285-03	19652.8
3.899990-02	8.912528-02	6.424993-03	19685.0
3.999990-02	8.809461-02	6.519926-03	19712.6
4.099989-02	8.699397-02	6.614471-03	19761.3
4.199989-02	8.578266-02	6.709677-03	19842.9
4.299988-02	8.458744-02	6.805525-03	19914.0
4.399988-02	8.340847-02	6.901886-03	19974.3
4.499987-02	8.224589-02	6.998632-03	20023.5

T	YU	SU	RDPL
4.599987-02	8.103697-02	7.095700-03	20084.5
4.699986-02	7.987072-02	7.193024-03	20124.1
4.799986-02	7.875835-02	7.290365-03	20137.6
4.899985-02	7.766285-02	7.387576-03	20138.7
4.999985-02	7.658434-02	7.484534-03	20127.4
5.099984-02	7.552264-02	7.580891-03	20105.0
5.199983-02	7.440378-02	7.676135-03	20103.2
5.299983-02	7.325686-02	7.770377-03	20110.1
5.399982-02	7.207442-02	7.863614-03	20129.1
5.499982-02	7.090742-02	7.955839-03	20139.0
5.599981-02	6.976001-02	8.046973-03	20137.8
5.699981-02	6.863220-02	8.136939-03	20125.3
5.799980-02	6.752399-02	8.225663-03	20101.4
5.899980-02	6.634303-02	8.313131-03	20107.4
5.999979-02	6.511692-02	8.399523-03	20130.8
6.099979-02	6.391360-02	8.484711-03	20141.4
6.199978-02	6.273293-02	8.568479-03	20139.8
6.299977-02	6.153988-02	8.650782-03	20142.7
6.399977-02	6.035593-02	8.731633-03	20139.4
6.499976-02	5.922697-02	8.810946-03	20107.0
6.599976-02	5.813685-02	8.888589-03	20053.4
6.699975-02	5.697397-02	8.964559-03	20035.8
6.799975-02	5.574999-02	9.039088-03	20048.2
6.899974-02	5.455266-02	9.112147-03	20044.6
6.999974-02	5.338195-02	9.183662-03	20024.6
7.099973-02	5.223779-02	9.253546-03	19988.3
7.199973-02	5.104416-02	9.321771-03	19978.8
7.299972-02	4.987319-02	9.388369-03	19955.0
7.399972-02	4.869692-02	9.453296-03	19933.2
7.499971-02	4.750686-02	9.516609-03	19918.2
7.599970-02	4.634716-02	9.578267-03	19883.1
7.699970-02	4.507836-02	9.638318-03	19916.5
7.799969-02	4.380430-02	9.696937-03	19952.0
7.899969-02	4.256573-02	9.754042-03	19962.6
7.999968-02	4.130232-02	9.809605-03	19989.5
8.099968-02	4.007057-02	9.863612-03	19992.8
8.199967-02	3.887628-02	9.915963-03	19967.3
8.299967-02	3.757739-02	9.966698-03	20021.0
8.399966-02	3.628307-02	1.001597-02	20070.4
8.499966-02	3.503239-02	1.006368-02	20083.6
8.599965-02	3.379190-02	1.010975-02	20087.8
8.699964-02	3.252352-02	1.015422-02	20115.1
8.799964-02	3.130282-02	1.019705-02	20097.9
8.899963-02	3.012965-02	1.023814-02	20034.3
8.999963-02	2.894403-02	1.027741-02	19982.5

T	YU	SU	RDPL
9.099962-02	2.768702-02	1.031505-02	20000.6
9.199962-02	2.648525-02	1.035101-02	19959.1
9.299961-02	2.533849-02	1.038519-02	19855.4
9.399961-02	2.424632-02	1.041750-02	19687.7
9.499960-02	2.308729-02	1.044793-02	19600.4
9.599960-02	2.187630-02	1.047673-02	19572.8
9.699959-02	2.073169-02	1.050386-02	19457.0
9.799958-02	1.965292-02	1.052919-02	19249.8
9.899958-02	1.863913-02	1.055265-02	18950.3
9.999957-02	1.768916-02	1.057419-02	18560.4
.101000	1.680154-02	1.059381-02	18085.1
.102000	1.679472-02	1.061092-02	16430.5
.103000	1.679389-02	1.062534-02	15028.3
.104000	1.679189-02	1.063761-02	13837.2
.105000	1.678904-02	1.064812-02	12816.9
.106000	1.678557-02	1.065720-02	11936.7
.107000	1.678168-02	1.066508-02	11172.3
.108000	1.677749-02	1.067197-02	10504.6
.109000	1.677313-02	1.067802-02	9918.48
.110000	1.676866-02	1.068335-02	9401.58
.111000	1.676416-02	1.068806-02	8943.88
.112000	1.675968-02	1.069225-02	8537.16
.113000	1.675526-02	1.069599-02	8174.60
.113999	1.675093-02	1.069933-02	7850.55
.114999	1.674672-02	1.070231-02	7560.24
.115999	1.674264-02	1.070499-02	7299.69
.116999	1.673872-02	1.070740-02	7065.50
.117999	1.673496-02	1.070957-02	6854.79
.118999	1.673138-02	1.071152-02	6665.11
.119999	1.672799-02	1.071327-02	6494.33
.120999	1.672478-02	1.071485-02	6340.66
.121999	1.672178-02	1.071626-02	6202.52
.122999	1.671897-02	1.071754-02	6078.58
.123999	1.671636-02	1.071867-02	5967.66
.124999	1.671397-02	1.071969-02	5868.77
.125999	1.671177-02	1.072058-02	5781.04
.126999	1.670979-02	1.072138-02	5703.72
.127999	1.670802-02	1.072207-02	5636.17
.128999	1.670646-02	1.072266-02	5577.84
.129999	1.670511-02	1.072317-02	5528.29
.130999	1.670397-02	1.072359-02	5487.12
.131999	1.670305-02	1.072393-02	5454.02
.132999	1.670233-02	1.072419-02	5428.75
.133999	1.670183-02	1.072437-02	5411.12
.134999	1.670154-02	1.072447-02	5401.00

Table 21. Optimal Control Law of Zone 8.

T (sec)	YU (in ²)	SU (in ²)	RDPL (lbf)
1.000000-03	9.771953-02	1.091185-02	4288.68
2.000000-03	9.780086-02	1.090183-02	4711.27
3.000000-03	9.809796-02	1.067211-02	6422.71
4.000000-03	9.836975-02	9.108397-03	10589.0
4.999999-03	9.921202-02	6.301377-03	15805.4
5.999998-03	9.799876-02	4.057114-03	20275.3
6.999998-03	9.772919-02	2.690429-03	22999.4
7.999997-03	9.921702-02	1.942810-03	23716.5
8.999996-03	.101811	1.997599-03	22741.0
9.999995-03	.102877	1.682034-03	22931.5
1.099999-02	.103619	1.516334-03	22967.1
1.199999-02	.103886	1.415936-03	23068.8
1.299999-02	.104024	1.354569-03	23141.1
1.399999-02	.104181	1.327430-03	23141.4
1.499999-02	.104161	1.329888-03	23149.3
1.599999-02	.103974	1.358639-03	23166.7
1.699998-02	.103588	1.410476-03	23211.5
1.799998-02	.103063	1.482900-03	23266.6
1.899998-02	.102550	1.573111-03	23282.6
1.999998-02	.101998	1.678288-03	23283.0
2.099997-02	.101231	1.796906-03	23334.1
2.199997-02	.100544	1.926947-03	23330.9
2.299997-02	9.979387-02	2.067927-03	23326.8
2.399997-02	9.905751-02	2.215548-03	23300.9
2.499996-02	9.811538-02	2.370068-03	23335.2
2.599996-02	9.714736-02	2.530817-03	23363.3
2.699996-02	9.624149-02	2.696521-03	23353.0
2.799996-02	9.532682-02	2.865941-03	23333.4
2.899995-02	9.430488-02	3.038563-03	23344.1
2.999995-02	9.323090-02	3.213897-03	23364.5
3.099995-02	9.217204-02	3.390143-03	23373.2
3.199994-02	9.125073-02	3.564442-03	23325.7
3.299994-02	9.029094-02	3.736332-03	23293.8
3.399993-02	8.924612-02	3.906238-03	23295.9
3.499993-02	8.815781-02	4.074380-03	23315.1
3.599992-02	8.708302-02	4.240767-03	23327.9
3.699992-02	8.602171-02	4.405329-03	23334.1
3.799991-02	8.488124-02	4.568072-03	23373.8
3.899990-02	8.362344-02	4.729567-03	23463.9
3.999990-02	8.238280-02	4.889862-03	23545.7
4.099989-02	8.115990-02	5.049435-03	23616.3
4.199989-02	7.993217-02	5.209329-03	23682.6
4.299988-02	7.867686-02	5.369549-03	23755.0
4.399988-02	7.750629-02	5.529795-03	23779.9
4.499987-02	7.636028-02	5.689674-03	23785.8

T	YU	SU	RDPL
4.599987-02	7.523350-02	5.848964-03	23775.0
4.699986-02	7.412593-02	6.007447-03	23747.2
4.799986-02	7.297160-02	6.164977-03	23735.9
4.899985-02	7.177399-02	6.321721-03	23739.0
4.999985-02	7.053749-02	6.477623-03	23754.7
5.099984-02	6.932144-02	6.632240-03	23753.8
5.199983-02	6.812760-02	6.784694-03	23737.5
5.299983-02	6.695589-02	6.934822-03	23705.6
5.399982-02	6.579583-02	7.082492-03	23663.9
5.499982-02	6.449500-02	7.227951-03	23698.3
5.599981-02	6.322016-02	7.371337-03	23714.3
5.699981-02	6.197130-02	7.512483-03	23711.3
5.799980-02	6.072028-02	7.651258-03	23706.0
5.899980-02	5.947010-02	7.787659-03	23696.2
5.999979-02	5.828068-02	7.921509-03	23644.4
6.099979-02	5.713544-02	8.052420-03	23561.5
6.199978-02	5.587733-02	8.180255-03	23550.2
6.299977-02	5.458840-02	8.305431-03	23556.5
6.399977-02	5.332988-02	8.427836-03	23540.2
6.499976-02	5.210168-02	8.547336-03	23500.8
6.599976-02	5.085817-02	8.663846-03	23470.2
6.699975-02	4.960219-02	8.777492-03	23445.9
6.799975-02	4.835880-02	8.888186-03	23410.2
6.899974-02	4.708426-02	8.996008-03	23395.3
6.999974-02	4.584464-02	9.100903-03	23351.0
7.099973-02	4.447897-02	9.202946-03	23405.2
7.199973-02	4.311748-02	9.302386-03	23454.5
7.299972-02	4.178116-02	9.399054-03	23481.8
7.399972-02	4.042983-02	9.492956-03	23520.9
7.499971-02	3.912157-02	9.583988-03	23519.4
7.599970-02	3.777963-02	9.672029-03	23549.3
7.699970-02	3.636289-02	9.757444-03	23649.8
7.799969-02	3.499715-02	9.840140-03	23698.4
7.899969-02	3.365299-02	9.919905-03	23723.8
7.999968-02	3.227278-02	9.996830-03	23786.8
8.099968-02	3.094846-02	1.007079-02	23784.7
8.199967-02	2.967983-02	1.014155-02	23714.4
8.299967-02	2.833193-02	1.020907-02	23743.2
8.399966-02	2.697923-02	1.027369-02	23775.0
8.499966-02	2.569174-02	1.033518-02	23716.2
8.599965-02	2.446905-02	1.039330-02	23562.4
8.699964-02	2.321259-02	1.044793-02	23460.8
8.799964-02	2.186189-02	1.049958-02	23500.1
8.899963-02	2.059081-02	1.054814-02	23405.3
8.999963-02	1.939876-02	1.059334-02	23169.3

T	YU	SU	RDPL
9.099962-02	1.828466-02	1.063499-02	22789.8
9.199962-02	1.724697-02	1.067293-02	22270.0
9.299961-02	1.689450-02	1.070656-02	20507.0
9.399961-02	1.690448-02	1.073445-02	18438.7
9.499960-02	1.691105-02	1.075768-02	16721.6
9.599960-02	1.691506-02	1.077725-02	15279.7
9.699959-02	1.691713-02	1.079388-02	14057.1
9.799958-02	1.691774-02	1.080812-02	13011.5
9.899958-02	1.691723-02	1.082042-02	12110.4
9.999957-02	1.691589-02	1.083109-02	11328.6
.101000	1.691391-02	1.084041-02	10646.3
.102000	1.691145-02	1.084860-02	10047.7
.103000	1.690866-02	1.085581-02	9519.98
.104000	1.690562-02	1.086220-02	9052.86
.105000	1.690242-02	1.086788-02	8637.85
.106000	1.689913-02	1.087293-02	8267.93
.107000	1.689580-02	1.087746-02	7937.28
.108000	1.689246-02	1.088150-02	7641.02
.109000	1.688916-02	1.088514-02	7375.06
.110000	1.688593-02	1.088841-02	7135.91
.111000	1.688278-02	1.089135-02	6920.63
.112000	1.687974-02	1.089400-02	6726.70
.113000	1.687681-02	1.089638-02	6551.96
.113999	1.687402-02	1.089853-02	6394.55
.114999	1.687138-02	1.090046-02	6252.88
.115999	1.686889-02	1.090220-02	6125.57
.116999	1.686655-02	1.090375-02	6011.42
.117999	1.686439-02	1.090514-02	5909.41
.118999	1.686239-02	1.090638-02	5818.64
.119999	1.686057-02	1.090747-02	5738.35
.120999	1.685893-02	1.090843-02	5667.87
.121999	1.685747-02	1.090926-02	5606.65
.122999	1.685619-02	1.090998-02	5554.20
.123999	1.685510-02	1.091058-02	5510.12
.124999	1.685419-02	1.091107-02	5474.09
.125999	1.685347-02	1.091145-02	5445.83
.126999	1.685294-02	1.091173-02	5425.14
.127999	1.685259-02	1.091191-02	5411.87
.128999	1.685244-02	1.091200-02	5405.93